Financing AI Infrastructure: Growth and Technological Leadership

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October 2026

Abstract

AI infrastructure investment generates benefits that extend beyond investors and national borders. We develop a two-country growth model of infrastructure finance and competing AI laboratories. Sufficiently large labor-income gains that investors cannot capture can make tax-backed construction optimal despite negative private net returns. When deployment feedback benefits a foreign laboratory, investment can raise domestic welfare and world growth while reducing domestic technological leadership. Enforceable construction grants can implement investment under private control. The analysis shows how income claims and research feedback shape the divergence between private investment incentives, national welfare, and technological leadership.

Keywords: artificial intelligence; infrastructure; innovation; public finance; investment incentives; technological leadership.

JEL classification: O31, O38, H54, G32, F43.

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Xia acknowledges support from the National Natural Science Foundation of China (72595873; 72473172).

1 Introduction

Artificial intelligence (AI) has made infrastructure investment a central part of technological competition. Training and operating models require computing facilities, reliable electricity, and connections to users. The United States and China accounted for an estimated 45 and 25 percent, respectively, of global data-center electricity consumption in 2024 (International Energy Agency, 2025). Both countries are expanding the infrastructure that supports AI, with different roles for public coordination and corporate investment.

China's approach gives public coordination a prominent role. The 2026 Government Work Report calls for large AI-computing clusters and coordinated development of computing and electricity (State Council, 2026). The National Development and Reform Commission estimated that investment in six infrastructure networks and related priority fields would exceed RMB 7 trillion in 2026 (NDRC, 2026). Power, computing, and communications are three of these networks. This broad program combines public and private funding across infrastructure sectors.

In the United States, large technology companies are major sponsors of AI infrastructure investment. Amazon reported global cash capital expenditures of $128.3 billion in 2025, while Meta reported $72.22 billion including finance-lease principal payments (Amazon, 2026; Meta, 2026). Corporate projects also draw on external investors: Meta's Hyperion campus combines a joint venture, private debt, and facility leases (Meta, 2025b). Federal policy supports permitting and land access and calls for financing support (The White House, 2025).

Table 1 summarizes the scale and organization of these investments. Panel A documents the growth in Amazon's and Meta's company-wide capital expenditure. Panel B reports China's broader infrastructure program and Alibaba's AI and cloud investment plan, with their respective scopes and horizons. These arrangements allocate project revenues, investment authority, and financing obligations differently. AI infrastructure also generates information useful for future research and productivity gains for users and workers. The recipients of these benefits need not be the infrastructure sponsor. These differences motivate a comparison between corporate investment and tax-backed public investment. When will corporate investors undertake nationally desirable projects? How does public finance affect investment and the international distribution of technological gains?

We develop a two-country dynamic economy in which AI laboratories compete to improve a common quality frontier. Training and inference share electricity and computing resources, and ordinary production also uses electricity. Deployment generates opportunities to validate information that improves later research. Feedback entitlements determine which laboratory can use this information, while commercial authorization determines where the current model can be used. The frontier model earns international royalties. Knowledge embodied in earlier generations also dif-

Table 1. Corporate capital expenditure and Chinese infrastructure plans

A. Reported corporate expenditure (USD billion)

Company	2024	2025	Coverage and accounting measure
Amazon	77.7	128.3	Global cash capital expenditure; technology infrastructure, principally AWS, and fulfillment capacity.
Meta	39.23	72.22	Global capital expenditure, including principal payments on finance leases.

B. Announced Chinese infrastructure investment

Program or company	Amount	Horizon and coverage
Six networks and related priority fields	Above RMB 7 trillion	Anticipated investment during 2026 across public and private funding sources, announced by the Chinese government in March 2026.
Alibaba	At least RMB 380 billion	AI and cloud infrastructure over the three years following its February 2025 announcement.

Note: Panel A reports company-wide realized expenditure. Amazon's cash measure is net of property and equipment sales and incentives; Meta includes finance-lease principal. Panel B reports plans with their stated horizons. The six-network estimate also covers transport, public services, and other priority fields. Each entry retains its own currency and expenditure scope. *Sources:* Amazon (2026), Meta (2025a, 2026), NDRC (2026), and Alibaba Group (2025).

fuses into general production and raises labor productivity. Thus technical progress generates both corporate distributions and labor income, with distinct implications for infrastructure finance.

One country has an infrastructure expansion opportunity. We compare corporate investment control with national control, holding the physical economy and initial resources fixed and preserving independent control of laboratory research. Corporate sponsors can rearrange the available domestic corporate income claims and borrow against enforceable receipts. A national government values domestic consumption and can issue claims backed by future taxes, subject to fiscal feasibility and real collection costs. Both governments reconsider commercial authorization as technological leadership and research information change. Investment, research, access, and financing are determined in stationary Markov-perfect equilibrium, with enforceable fiscal obligations carried forward after issuance.

The central result is that nationally desirable AI investment can reduce domestic technological leadership. In a nonempty region of primitives, every admissible allocation of domestic corporate cash-flow claims leaves construction unattractive, even when credit is unconstrained. The national government nevertheless builds immediately using fairly priced fiscal claims. Construction raises domestic welfare and global growth, while reducing both the domestic share of inventions and the fraction of time the domestic laboratory owns the frontier. Both incumbent and challenger laboratories optimally conduct research. The comparison persists in a local extension with strictly interior research choices, including cases in which both laboratories increase their effort after construction.

The mechanism combines the allocation of research feedback with the incidence of growth income. When useful feedback from additional domestic deployment accrues abroad, it improves the foreign laboratory's research productivity. The resulting technical progress also benefits domestic workers. Corporate contracts in the model assign firms' cash flows, while workers retain the labor-income gains generated by technical progress. The national investment objective includes these gains. Public support becomes attractive when this omitted income gain covers the remaining engineering and fiscal costs and the value of postponement. Tax-backed finance then makes the desired construction date feasible. The result therefore requires both an investment incentive and an enforceable means of paying for it. The equilibrium conditions also ensure that commercial access and tax collection remain optimal after construction and inherited obligations can be honored.

The allocation of feedback determines the technological beneficiary. Local feedback can strengthen domestic research; feedback assigned abroad can shift the gains toward the foreign laboratory. Income incidence also depends on ownership and the diffusion of model knowledge. We distinguish the rate of worldwide technical progress, a country's share of inventions, and its share of time at the frontier. Invention shares and frontier ownership shares can respond differently because innovation rates depend on who currently leads and what information each laboratory retains. This distinction makes it possible to assess technological competition without using frontier ownership as a proxy for national welfare.

A numerical illustration examines the mechanism with conventional quadratic research costs, positive feedback exposures for both laboratories, and fixed labor endowments and construction cost. Varying only the allocation of feedback from domestic deployment changes the distribution of its gains. The results include cases in which public investment raises domestic research and the domestic innovation rate while lowering both domestic leadership shares. Other assignments produce opposite movements in invention shares and frontier ownership shares, or raise both. The transition also matters: construction absorbs resources immediately, whereas productivity benefits accumulate. A positive lifetime consumption gain can coexist with a loss over the first several model dates. The illustration thus connects the stationary comparison to research responses and the timing of national income, using a common set of hypothetical primitives.

We finally ask when fiscal support can implement the investment under private construction control. Income claims and pledgeability address different constraints. Full pledgeability can finance privately profitable construction. Capturing the omitted labor-income return additionally requires an income assignment or fiscal transfer. An enforceable construction grant can supply both the missing incentive and the required funding. We characterize the minimum nonnegative Markov grant schedule when investors can wait for future support. A future grant needed to overcome a funding constraint can leave the investor a rent, raising the support required for immediate construction. Private implementation is available when this schedule is fiscally feasible and the government

continues to prefer providing it. The institutional comparison consequently identifies conditions under which public resources are valuable and conditions under which public construction control can be replaced by a supported private investor.

1.1 Related literature

This paper relates to four strands of literature. First, models of data and innovation connect economic activity to research. Farboodi and Veldkamp (2026) model data generated by transactions, while Morellec and Zucchi (2026) connect production-generated data, computing, and quality-ladder research. Data ownership affects nonrival input use in Jones and Tonetti (2020), and data-enabled learning shapes competitive advantage in Hagiu and Wright (2023). Farboodi, Koh, and Xia (2026) show how automation's wage effects depend on task substitution and data spillovers. We connect these mechanisms to infrastructure investment by separating the sponsor, the laboratory receiving deployment feedback, and the households receiving growth income. Local capacity determines how much AI can be deployed, while feedback rights determine which laboratory learns from that use. We show how domestic deployment can improve foreign research incentives while domestic workers share in the resulting growth.

Second, the institutional analysis draws on ownership and investment incentives (Grossman and Hart, 1986), limits to debt capacity (Hart and Moore, 1994), and public liquidity provision (Holmström and Tirole, 1998). Malamud and Zucchi (2019) connect liquidity to innovation and growth. Itskhoki and Moll (2019) characterize development policies that accelerate entrepreneurial wealth accumulation and relax future borrowing constraints. Choi and Levchenko (2025) study how temporary credit subsidies produce persistent productivity gains through learning by doing. Kogan et al. (2020) show that innovating firms' shareholders need not capture all innovation gains. In our model, pooling domestic corporate cash flows leaves labor-income gains outside investors' claims. We identify nationally desirable construction that remains unprofitable under every admissible corporate pooling arrangement, even with unconstrained credit. The allocation of growth income thus creates an investment incentive gap that additional lending alone cannot eliminate.

Third, international innovation models connect research incentives to market opportunities and comparative advantage (Grossman and Helpman, 1991; Somale, 2021; Sampson, 2023). Fornaro and Wolf (2026) study tariffs, technological rents, and innovation; Cesa-Bianchi et al. (2026) relate industrial policy, capital flows, and technological leadership. We show how infrastructure-supported deployment changes research competition through the international allocation of feedback. In the resulting investment equilibrium, tax-backed construction can raise domestic welfare and world growth while reducing the financing country's invention and frontier ownership shares.

The national return to infrastructure can therefore diverge from its contribution to domestic technological leadership.

Finally, our policy analysis relates to innovation support and AI investment. Akcigit et al. (2022) study innovation policy under spillovers and private information. Evidence on public R&D spending documents persistent productivity effects (Antolin-Diaz and Surico, 2025) and international research spillovers (Moretti et al., 2025). Recent AI finance research examines the organization of infrastructure investment (Van Nieuwerburgh, 2026) and strategic investment with excess capacity and financial fragility (Rungcharoenkitkul, 2026). With contractible construction, we characterize the minimum Markov grant schedule that addresses both return and funding requirements when investors can wait for future support. The resulting waiting rents can raise the grant required for immediate construction. Fiscal feasibility and government continuation incentives identify when public support can implement construction under private control.

The rest of the paper is organized as follows. Section 2 presents the economy. Section 3 studies how deployment changes international innovation, and Section 4 establishes the national investment and leadership result. Section 5 examines endogenous research responses, income incidence, and the transition. Section 6 characterizes fiscal support and private implementation. Section 7 concludes.

2 The economy

There are two countries, indexed by $i \in \{1, 2\}$, and an infinite sequence of dates $t = 0, 1, \ldots$. Each country contains households, competitive producers, an independently controlled research laboratory, and a government. An infrastructure owner supplies electricity and installed computing capacity. Country 1 has an opportunity to expand its infrastructure; country 2's infrastructure is installed. The two laboratories compete to produce the next generation of a model that can be licensed in both markets. The institutional comparison concerns country 1's investment opportunity within this bilateral research and commercial-access economy.

2.1 Households, ownership, and technology

Households supply L_i units of labor inelastically and have preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t C_{it}, \qquad 0 < \beta < 1. \tag{1}$$

They own domestic firms, fixed factors, and financial claims. Financial claims are held domestically. Corporate investors maximize their shareholders' discounted distributions, whereas the government maximizes the sum of its residents' utilities. Corporate agreements can combine income claims across domestic firms while preserving the laboratory's independent research control.

The frontier model has quality $Q_t > 0$ and a commercial owner $\ell_t \in \{1, 2\}$. The predecessor, of quality Q_t/a, is freely available, where $a > 1$. An innovation raises quality by the factor a:

$$Q_{t+1} = a^{\zeta_t} Q_t, \qquad \zeta_t \in \{0, 1\}, \qquad \beta a < 1. \tag{2}$$

The successful laboratory becomes the frontier owner, and the previous frontier becomes the freely available predecessor. A laboratory retains its identity after losing the frontier and can innovate while incumbent. It therefore values future model rents and internalizes replacement of its own current product.

Quality measures the productivity of economically useful model services. The final good is freely traded and is the numeraire. A competitive background sector produces Q_t goods per unit of labor, so the wage is Q_t whenever this sector operates. The background productivity Q_t represents knowledge embodied in freely available earlier model generations. This knowledge improves ordinary work without requiring a new inference for each unit of labor. Its diffusion continues when commercial access to the current frontier is restricted. The wage spillover gives households a stake in technological progress alongside the returns earned by corporate owners.[1] The restriction $\beta a < 1$ makes discounted values finite for every feasible innovation history.

Two domestic activities use electricity. An ordinary sector produces $Q_t b_i \sqrt{m_{it}}$ with electricity m_{it} and a domestically owned fixed factor, where $b_i > 0$. An AI application uses one unit of electricity and one unit of compute per unit of inference e_{it}. Its output is $Q_t D_i e_{it}$ with the frontier and $Q_t D_i e_{it}/a$ with the predecessor, where $D_i > 0$ is output per inference. The concavity of ordinary production gives electricity a competing use.[2] Applications requiring local execution are served by domestic capacity; examples include installations with material latency, adaptation, or confidentiality requirements.

Country 1's capacity is $k_t \in \{L, H\}$. Its electricity endowment is $E_1(k_t)$ and its compute endowment is $K_1(k_t)$, each weakly larger at H. Country 2 has fixed (E_2, K_2). A construction decision $I_t \in \{0, 1\}$ is available at $k_t = L$. It costs $Q_t J$ units of the final good, $J > 0$, and makes the higher capacity available at $t + 1$. High capacity is durable. A public construction order can also use $Q_t c_G$ goods for implementation, with a bounded inherited-state-dependent cost $c_G \geq 0$; $c_G = 0$

[1] In customer support, Brynjolfsson et al. (2025) document productivity gains from AI assistance and evidence of worker learning.

[2] Meeks et al. (2026) study how data-center entry affects grid investment and electricity costs for other users.

under corporate construction and in the numerical illustration. All institutions face this engineering technology and delivery lag; their investment decisions determine when capacity arrives.

2.2 Training, inference, and the operating equilibrium

Laboratory i chooses research effort $r_i \in [0, \bar{r}_i]$, using $\mathcal{L}_i(r_i)$ units of labor and $\varepsilon_T r_i$ units each of electricity and compute, with $\mathcal{L}_i(0) = 0$, $\mathcal{L}_i$ continuously differentiable, increasing, and convex, and $\varepsilon_T > 0$. Training and deployment clear the same input markets:

$$m_i + \varepsilon_T r_i + e_i \le E_i, \qquad \varepsilon_T r_i + e_i \le K_i. \tag{3}$$

A trained model can be reused in another country without repeating its development cost; each inference requires locally supplied inputs that are rival in use. This distinction connects a geographically transferable research asset to local infrastructure.

The frontier owner's government chooses whether to authorize foreign commercial use during the date. Write $o_i = 1$ when country i has frontier access and $o_i = 0$ when it uses the predecessor. Domestic access is always authorized. Access is renewed each date, and the owner of an authorized commercial right supplies the model under a standardized service contract. Its royalty per inference is $Q_t \mu D_i$, where $\mu = 1 - 1/a$. At this royalty the user is indifferent between the frontier and the predecessor; it uses the frontier when access is authorized. The application sector's net value of an input bundle is consequently $Q_t p_i$, where $p_i = D_i/a$. The royalty assigns the current model's quality increment to its owner. Under this service obligation, government authorization determines current commercial access.

Electricity and compute are competitively rented at prices $Q_t u_i$ and $Q_t v_i$. Put $m_i^* = (b_i/(2p_i))^2$. We maintain an active operating region in which every feasible research choice leaves positive inference and positive background employment. Profit maximization and market clearing then yield

$$\begin{aligned} m_i &= \max\{m_i^*, E_i - K_i\}, \quad & e_i &= E_i - \varepsilon_T r_i - m_i, \\ u_i &= \frac{b_i}{2\sqrt{m_i}}, & v_i &= p_i - u_i \ge 0. \end{aligned} \tag{4}$$

When compute is slack, $m_i = m_i^*$ and $v_i = 0$. Otherwise both capacity constraints bind and $m_i = E_i - K_i$. The relevant restrictions are $m_i < E_i - \varepsilon_T \bar{r}_i$ and adequate labor for all research and financing obligations. These restrictions apply to deviations as well as equilibrium actions. In this region $u_i + v_i = p_i$, so the unit training input cost is $p_i \varepsilon_T$, independently of capacity. Infrastructure affects research through model revenues and useful deployment feedback. The linear application

technology and common training-input proportions keep the unit input price fixed on this operating region.

Define effective inference capacity before training by

$$\bar{e}_i = \min\{K_i, E_i - m_i^*\}, \qquad e_i = \bar{e}_i - \varepsilon_T r_i. \tag{5}$$

Holding research fixed, deployment expands when investment relaxes the binding capacity constraint. Achieving a given deployment target requires sufficient electricity and computing capacity. When computing capacity binds, a compute expansion with fixed power diverts electricity from ordinary production; a joint expansion can raise both inference and ordinary output. Appendix B derives the marginal returns to each input and their distribution across infrastructure and ordinary producers.

The competitive application sector earns zero operating profit after paying for model services and physical inputs: $\pi_i^{\text{app}} = Q_t(p_i - u_i - v_i)e_i = 0$. Ordinary-sector profit Π_i accrues to its fixed factor. The normalized receipts of the infrastructure owner and the profits of the ordinary fixed factor are

$$R_i(k) = u_i E_i(k) + v_i K_i(k), \qquad \Pi_i(k) = b_i\sqrt{m_i} - u_i m_i. \tag{6}$$

They are independent of current research and access in this active region. Infrastructure receives the scarcity rents on its assets; ordinary producers retain the complementary return to their fixed factor. Ordinary-sector profits are among the income claims that corporate sponsors can acquire in Section 4.

The laboratory's normalized current royalty income and net distribution are

$$\begin{aligned} P_i(\ell, o, r) &= \mathbf{1}_{\{\ell=i\}}\mu \sum_{j=1}^{2} o_j D_j e_j, \\ \pi_i^R(\ell, o, r) &= P_i(\ell, o, r) - \mathcal{L}_i(r_i) - p_i\varepsilon_T r_i. \end{aligned} \tag{7}$$

Domestic shareholders fund research and receive the laboratory's net distributions. More research can raise the probability of the next model, but it also consumes inputs that could support current application output, model royalties, and application experience.

2.3 Research and usable application feedback

Let $d_i \in \{0, 1\}$ indicate whether laboratory i has validated application information available for research. Baseline research productivity is $\chi_i > 0$, and the productivity gain from that feedback is

$\nu_i \geq 0$. Research has effective intensity $h_i = \chi_i(1 + \nu_i d_i)r_i$. The probabilities of no innovation and of an innovation by country i are

$$\Pr(\zeta = 0) = \frac{1}{1 + h_1 + h_2}, \qquad \Pr(\zeta = 1, \ell' = i) = \frac{h_i}{1 + h_1 + h_2}. \tag{8}$$

This contest selects at most one successful new generation per date and has diminishing returns to own effort at a fixed rival effort. Baseline research productivity and useful application information are separate sources of innovative capacity. A country with lower baseline research productivity can have a larger gain from learning through deployment.

Application experience becomes a research input after validation. Let $\Lambda_{ji} \geq 0$ be laboratory j's usable exposure per unit of inference in country i, combining information quality and the right to use it. Define $w_i(o_i) = o_i + (1 - o_i)\vartheta_i$, where $\vartheta_i \in [0, 1]$ measures the research relevance of predecessor applications. Conditional on no innovation, a laboratory without validated feedback acquires it with probability

$$f_j = \frac{Z_j}{1 + Z_j}, \qquad Z_j = \sum_{i=1}^{2} \Lambda_{ji} w_i(o_i) e_i. \tag{9}$$

The columns of Λ identify where applications operate; its rows identify the laboratories that can use their experience. Diagonal entries represent local learning, and off-diagonal entries represent cross-border learning. The commercial license and the feedback entitlement are separate contracts.[3]

An existing validated set depreciates with probability $\delta_j \in (0, 1)$ on a date without innovation and survives a new model generation with probability $\omega \in [0, 1]$:

$$\begin{aligned} \Pr(d'_j = 1 \mid \zeta = 0, d_j, e, o) &= (1 - d_j) f_j + d_j(1 - \delta_j), \\ \Pr(d'_j = 1 \mid \zeta = 1, d_j) &= \omega d_j. \end{aligned} \tag{10}$$

Conditional on the innovation outcome and current actions, the feedback draws are independent across laboratories. Complete reset corresponds to $\omega = 0$; positive retention allows application knowledge to remain useful across model generations. Together with construction, (8)–(10) determine the transition law.

For local feedback it is useful to write $\Lambda_{ii} = \bar{\lambda}_i \sigma_i$, where $\bar{\lambda}_i > 0$ measures potentially informative exposure and $\sigma_i \in [0, 1]$ is its legally usable fraction. We also write $\lambda_i = \Lambda_{ii}$ in the local-learning specialization. Expansion raises research productivity through the exposure received by the learn-

[3]For example, business users of OpenAI services retain control over whether their information is used for model training (OpenAI, 2026). The model treats feedback entitlements as given contracts.

Table 2. The boundary of admissible private organization

Right or arrangement	Treatment in the corporate comparison
Infrastructure and ordinary profits	Transferable subject to acquisition payments and participation.
Laboratory distributions	Passive claims; the laboratory retains its independent research mandate.
Future household wages	Outside the set of enforceable corporate claims.
Household collective provision	Requires an additional household agreement; unilateral contributions are analyzed in Appendix G.4.
Construction grants and guarantees	Fiscal instruments, analyzed separately from unaided corporate aggregation.
Research or data control	Retained by the laboratory and application-data owners.

ing laboratory. Commercial authorization affects the quality of that exposure through w_i, while inherited validated information supports current research.

2.4 Income claims, financing, and fiscal capacity

The infrastructure investor receives the cash flows legally assigned to it. A separated infrastructure company receives $R_1(k)$; contracts with beneficiaries can add ordinary-sector profits and laboratory income. Laboratory claims carry both distributions and associated shareholder funding calls. They are nonvoting, and the laboratory continues to maximize its standalone enterprise value. Households retain their labor income. Corporate agreements cannot pledge future wages that households have not assigned. Shareholders supply funds for financial returns; contributions motivated by unassigned wage benefits and collective worker ownership require additional household agreements outside this corporate institution. Fiscal authority has a distinct, legally enforceable tax base. Table 2 summarizes these financing and control arrangements. The contract set permits the consolidation of corporate profits while leaving research decisions independent and household labor-income gains outside the corporate investment objective. This distinction determines which returns private organization can internalize before fiscal support is introduced.

Write S_k for the investor's assigned operating receipts, suppressing any dependence on frontier ownership and feedback. The financing institution permits retained earnings and construction-contingent sales of future income. Liquid resources w are measured in units of current quality. At construction, the investor can sell an enforceable fraction $\kappa^p \in [0, 1]$ of its unencumbered future distributions. Buyers receive the specified share of realized income, and competitive households price the claims using their common discount factor. Thus a risky revenue claim is priced using its state-contingent payments. Before construction, retained earnings are safe deposits with domestic households, earning the gross return $1/\beta$. Their counterpart is a household repayment obligation

backed by labor income. The bound on financial positions and adequate labor income ensure repayment. Deposit payments cancel in national accounts. The shareholder cannot be compelled to supply resources beyond these contracts. Section 6.1 derives the funding limit, and Appendix I.1 gives the retained-earnings dynamics.

Tax quantities are measured per unit of current quality. Current collection T provides net receipts $n(T)$ and uses $\Phi(T) = T - n(T) \geq 0$ units of goods in the same normalization. A public controller can apply net receipts to construction and can issue claims against legally assigned future receipts, subject to their enforceability and its intertemporal budget. Taxes and debt are domestic transfers; only $\Phi(T)$ is a resource cost. National control changes the investment objective, while tax backing expands the resources that can be pledged for repayment. Claims with identical payments have the same competitive price under public and private issuance.

A guarantee can relax a private pledgeability limit. Actual collection finances the residual requirement after project receipts have been capitalized. Section 4.2 derives that requirement and the least-cost collection path; Appendix H.4 gives the financing argument and its proof.

Financial choices must satisfy the dated budget, the enforceable-claim restriction, and the household and government no-Ponzi conditions. The state records outstanding claims as well as liquid assets, so the same future income cannot be pledged twice. For the retained-earnings formulation, issuance occurs at construction and subsequent laboratory and access policies depend on capacity and the operating state, independently of the distribution of these financial claims. The financing institution restricts liquid holdings to a bounded interval that includes states with sufficient cash to fund construction. Construction and saving choices respect this bound. Labor endowments are sufficiently large for positive background employment and for residents to meet all permitted financial calls.

2.5 Institutions, timing, and national accounts

Under corporate control, P, the infrastructure investor maximizes the value of its enforceable income claims. The compact set Θ of admissible arrangements includes a separated infrastructure company and contracts assigning ordinary profits and passive laboratory distributions. Each arrangement must satisfy participation and funding requirements. Under national control, G, the government maximizes domestic welfare, measured by expected discounted consumption, and can use costly tax backing. Corporate income remains domestically owned under both institutions. Laboratories independently maximize their enterprise values, and the physical project and initial national resources are common. Section 6 studies fiscal grants to privately controlled projects within the same environment.

At the start of a date, all players observe inherited capacity, frontier ownership, feedback and debt. Laboratories and governments submit their choices simultaneously, without observing the other submissions. Laboratories choose research, and the frontier owner's government chooses access. When a government controls infrastructure, it chooses access, construction, new debt and any tax jointly. Corporate financing takes place without observing the submitted research or access choices. The actions are then implemented, current markets operate and claims are settled. Innovation and feedback are realized, and newly ordered capacity becomes available next date. Thus a unilateral construction decision changes the continuation but cannot revise a laboratory's already chosen current research. For grant-financed private construction, Section 6.2 specifies the subsequent offer and acceptance stage.

Adding household and firm budgets cancels domestic lending, debt service, tax transfers, rebates and equity payments. Normalized national consumption is

$$c_i = L_i - \mathcal{L}_i(r_i) - (J + c_G)I\mathbf{1}_{\{i=1\}} - \Phi(T)\mathbf{1}_{\{i=1\}} + b_i\sqrt{m_i} + p_i e_i + P_i, \qquad C_i = Qc_i. \tag{11}$$

Infrastructure and ordinary-factor income enter this account once. Domestic corporate agreements redistribute those receipts. Cross-border royalty payments cancel in the world account:

$$\sum_i c_i = \sum_i \left[L_i - \mathcal{L}_i(r_i) + b_i\sqrt{m_i} + (p_i + o_i\mu D_i)e_i\right] - (J + c_G)I - \Phi(T). \tag{12}$$

Licensing therefore affects both the origin of model income and the quality of current applications. Construction and tax collection use real goods. With positive background employment, the welfare effect of investment reflects resource allocation and subsequent productivity.

2.6 Recursive equilibrium and technological leadership

The payoff-relevant aggregate state consists of quality Q, capacity, frontier ownership, feedback, and the financial balance sheet. Define $x = (\ell, d_1, d_2)$ and $s = (k, x, b)$, where b records liquid assets and all outstanding claims in units of current quality. Before construction, b records retained wealth w; after issuance, it records the assigned income shares and tax-payment obligations. We focus on stationary Markov strategies that are invariant to a common proportional change in quality and all quality-denominated claims. Under this restriction, strategies depend on s and value functions satisfy

$$V_j(Q, s) = Qv_j(s). \tag{13}$$

The homogeneity reduction retains every inherited variable affecting feasible policies or future payoffs.

For a joint action A, let $\mathcal{P}(ds', \zeta \mid s, A)$ denote the transition kernel and define

$$M(s, A, ds') = \beta \sum_{\zeta=0}^{1} a^{\zeta} \mathcal{P}(ds', \zeta \mid s, A). \tag{14}$$

The kernel combines discounting, quality growth, and state transitions; its total mass is at most $\beta a < 1$. At fixed operating policies that are independent of financial claims, it reduces to a finite matrix on x. On the full financial state space, integration replaces finite sums. Government i has flow payoff c_i, laboratory i has π_i^R, and a private infrastructure investor has its net distribution to shareholders.

Definition 1 (Stationary Markov-perfect equilibrium)**.** An equilibrium consists of stationary Markov strategies in the class specified above, value functions, prices, consumption, and allocations such that: (i) competitive firms optimize and markets clear; (ii) private and public budgets hold, contracted payments are enforceable, and claims are competitively priced; (iii) states obey the innovation, feedback, construction, and financial transition laws; and (iv) at every viable state each player maximizes

$$g_j(s, A_j, A_{-j}) + \int v_j(s') M(s, A_j, A_{-j}, ds'), \tag{15}$$

in expectation over opponents' strategies, and its value equals that maximum. Mixed strategies put weight only on maximizers. Corporate assignments satisfy the owners' participation constraints.

We study scale-invariant stationary Markov-perfect equilibria (Maskin and Tirole, 2001; Ericson and Pakes, 1995). Governments reoptimize access and investment after every history and honor inherited payment obligations. For fixed opponents on a bounded financial state space, each Bellman operator is a contraction with modulus at most βa. Bounded quality-scaled payoffs and the no-Ponzi conditions then give the one-stage deviation principle. Appendix A derives the household budgets, market clearing, and recursive valuation. The analysis below constructs equilibria by verifying the players' optimality conditions jointly.

Commercial authorization. At each history the frontier owner's government chooses whether to license its current model abroad. For fixed submitted research and a specified investment and financing choice, opening the foreign market raises its current royalties by $\mu D_j e_j$. Its one-date incentive is

$$\mu D_j e_j + \big[M(s, o_j = 1) - M(s, o_j = 0)\big] v_i \geq 0, \tag{16}$$

where v_i is its national continuation value on the full inherited state, including financing obligations. The kernel difference accounts for all changes in future validation and its national consequences.

Current royalties and domestic growth income can favor access even when foreign learning reduces domestic laboratory rents.

For any fixed profile, normalized values satisfy $v_j = (I - M)^{-1} g_j$, with the inverse interpreted as the convergent operator series $\sum_{t \geq 0} M^t$. Physical transitions use the different kernel $P(s, ds') = \sum_{\zeta} \mathcal{P}(ds', \zeta \mid s, A)$, averaged over equilibrium actions. From initial law μ_0, the state law at t is $\mu_0 P^t$. For a stationary ergodic recurrent operating process with invariant law π, define

$$\begin{aligned} q &= \int \Pr(\zeta = 1 \mid s)\pi(ds), \quad g = q \log a, \\ \theta_i &= \int \mathbf{1}_{\{\ell = i\}} \pi(ds), \qquad \rho_i = \frac{\int \Pr(\zeta = 1, \ell' = i \mid s)\pi(ds)}{q}. \end{aligned} \tag{17}$$

Here g is almost-sure log-quality growth, θ_i is the share of dates on which country i owns the frontier, and ρ_i is its share of new inventions when $q > 0$. We call θ_i the frontier ownership share. A larger share of time at the frontier can coexist with a smaller share of inventions. If normalized consumption c_{it} is bounded above and bounded away from zero, consumption in levels, $C_{it} = Q_t c_{it}$, grows at rate g in both countries on the recurrent class, even when their income levels and research positions differ. Write $W_i^{\mathcal{R}}(s)$ for national discounted consumption per unit of initial quality under institution $\mathcal{R}$. Institutional welfare comparisons hold initial assets, obligations, feedback, and frontier ownership fixed.

3 Deployment and international innovation

Investment changes both the scale of the model market and the local applications from which laboratories learn. Market expansion raises potential model revenues in both countries. The feedback effect accrues to the laboratories specified by Λ. World innovation and the investing country's frontier share are therefore different objects. We first give their stationary relationship, then characterize a branch in which both laboratories research in every state. For the analytical benchmark, set $\bar{r}_i = \eta$ and $\mathcal{L}_i(r) = \ell_i^r r$. Write $c_i^r = \ell_i^r + p_i \varepsilon_T$ for its resource cost, $\kappa_i = \mu D_i \varepsilon_T$ for royalties displaced by training, and $t_i(d_i) = \chi_i(1 + \nu_i d_i)$. Appendix D develops a local extension with interior research; Section 5 studies quadratic research costs.

3.1 Invention shares and time at the frontier

Table 3 distinguishes research inputs from measures of innovation and technological leadership. Let $q_i(x)$ be country i's current innovation probability and π the invariant operating distribution.

Table 3. Research activity and technological leadership

Object	Interpretation
$r_i(x)$	Research input in a given ownership and feedback state.
$\bar{q}_i$	Mean number of domestic frontier inventions per model date.
ρ_i	Domestic inventions as a share of all frontier inventions.
θ_i	Share of dates on which the domestic laboratory owns the frontier.

Its average number of inventions per date is $\bar{q}_i = \sum_x \pi(x) q_i(x)$, which we call its innovation rate, whereas $\rho_i = \bar{q}_i/(\bar{q}_1 + \bar{q}_2)$ and $g = (\bar{q}_1 + \bar{q}_2) \log a$.

The relationship between invention shares and frontier ownership shares holds across research policies. Let $q_i(s)$ and $q(s)$ be the conditional probabilities of an innovation by country i and of any innovation, and write $I_i(s) = \mathbf{1}_{\{\ell=i\}}$. In any ergodic recurrent equilibrium with $\bar{q} = E_\pi q(s) > 0$,

$$\rho_i - \theta_i = \frac{\mathrm{Cov}_\pi(q(s), I_i(s))}{\bar{q}}. \tag{18}$$

Indeed, $E[I_i(s') \mid s] = I_i(s)[1 - q(s)] + q_i(s)$. Stationarity implies $E_\pi[I_i q] = E_\pi q_i$, which gives (18). The formula permits state-dependent research, bilateral learning, and either retention or replacement of feedback. A country supplies a larger share of inventions than its share of frontier time precisely when innovation is more frequent in states in which it owns the frontier. The research equilibrium below identifies a feedback mechanism generating this association.

3.2 Incumbent innovation and persistent application knowledge

Both laboratories can innovate while incumbent and while challenger. A country can consequently produce successive generations, with research incentives reflecting current royalties and future creative destruction.

For an analytical benchmark, let $\nu_1 > 0$ and $\nu_2 = 0$. Country 1 can use validated application information to raise research productivity; country 2 relies on its baseline research productivity. Both laboratories still choose research, produce frontier innovations, and earn model income. The restriction isolates whether a local learning advantage can overturn a disadvantage in baseline research productivity.

Write $\mathcal{B}_k = \mu \sum_j D_j \bar{e}_j^k$ and $v_k = \mathcal{B}_k/(1-\beta)$. With linear research costs, a sufficient parameter restriction for research by both incumbents and challengers is

$$0 < c_i^r + \kappa_i < \beta(a-1)\chi_i v_L, \qquad i = 1, 2. \tag{19}$$

For every sufficiently small positive η, both laboratories strictly choose η at every ownership and feedback state under open access. We refer to this as the active-research branch. The same actions are optimal at permanent low capacity, permanent high capacity, and a low-capacity date followed by construction. Appendix C gives a finite-η sufficient bound that verifies every deviation in $[0, \eta]$. The region is nonempty: at any $1 < a < 1/\beta$, sufficiently small positive training coefficients and research labor costs satisfy (19). This restriction makes incumbent research strictly optimal in the quality-ladder contest.

Set $\Lambda_{11} = \lambda_1 > 0$ and all other feedback exposures to zero. Fix capacity on this branch and put

$$x = \eta\chi_1, \qquad y = \eta\chi_1(1 + \nu_1), \qquad z = \eta\chi_2, \qquad f = \frac{\lambda_1 e_1}{1 + \lambda_1 e_1}, \qquad e_1 = \bar{e}_1 - \varepsilon_T \eta, \tag{20}$$

and let $\delta = \delta_1$. Here x and y are the home laboratory's unvalidated and validated research intensities; z is the foreign intensity. These intensities apply in both incumbent and challenger roles.

Proposition 1 (Deployment and domestic technological leadership)**.** *On the open-access active-research branch, suppose $x, z > 0$, $y > x$, $0 < f < 1$, and $0 < \delta_1 < 1$. For every $\omega \in [0, 1]$, an increase in home inference or the home laboratory's usable feedback rights raises world growth, the home invention share, and the home frontier ownership share. With positive retention, the home invention share exceeds its frontier ownership share; the two shares coincide under complete reset.*

If $x < z < y$, home produces a majority of inventions precisely when

$$f > f^*_\rho := \frac{(z - x)\{\delta_1 + (1 - \omega)(y + z)\}}{y - z}. \tag{21}$$

A home frontier ownership share above one half has a unique positive threshold f^*_θ, with $f^*_\theta > f^*_\rho$ for $\omega > 0$. Each threshold is attainable through deployment when it is below one.

The proof solves the invariant distribution of frontier ownership and validated information. Appendix C gives the closed forms, the quadratic equation for f^*_θ, and the research optimality conditions.

The condition $x < z < y$ has a direct economic meaning: the home laboratory has lower baseline research productivity and can become more productive after validation. The research ranking consequently depends on how often the home laboratory has validated information. Infrastructure changes the frequency with which the home laboratory can use the more productive research technology. Data that survive a new generation lower the deployment needed to become the source of most inventions.

The comparison also separates the share of time at the frontier from the share of innovations. With complete reset, innovation cycles are independent of the previous winner. Each country can

win successive cycles, and its share of cycle winners equals its share of calendar time at the frontier. When useful data persist, home innovations occur disproportionately in the more research-intensive information state. Counting these advances gives home a larger share than weighting frontier ownership by calendar time. There is a nonempty region in which home produces most new inventions while owning the frontier on fewer than half of dates.

The threshold makes the content of the infrastructure advantage explicit. With the entitlement decomposition $\lambda_1 = \bar{\lambda}_1\sigma_1$, the fraction $\sigma_1 \in [0,1]$ determines how much local inference generates feedback legally available to the home laboratory. For either leadership criterion $j \in \{\rho, \theta\}$, if $\sigma_1 > 0$ and $0 < f_j^* < 1$, the necessary and sufficient deployment condition on this branch is

$$\bar{e}_1 > \varepsilon_T \eta + \frac{f_j^*}{\bar{\lambda}_1 \sigma_1 (1 - f_j^*)}. \tag{22}$$

Without an entitlement to useful application information, expanding local deployment cannot create this learning advantage. Electricity and compute must jointly meet this deployment requirement.[4]

Figure 1 displays the joint role of deployment and data access. Let $h_j = f_j^*/(1-f_j^*)$ for a feasible majority threshold. The two boundaries are $\sigma_1\bar{\lambda}_1 e_1 = h_\rho$ and $\sigma_1\bar{\lambda}_1 e_1 = h_\theta$, with $h_\theta > h_\rho$ under positive retention. A country below the lower boundary can expand inference and remain below the majority threshold if the laboratory captures too little usable feedback. Conversely, an improvement in data access reduces the additional capacity needed. The curves describe the deployment and feedback-entitlement combinations needed to reach each majority threshold.

Assigning country 1's feedback to the foreign laboratory reverses the incidence of the learning gain.

Corollary 1 (Domestic deployment and foreign research gains)**.** *Suppose useful feedback from country 1 is available to laboratory 2: $\Lambda_{21} > 0$, all other entries are zero, $\nu_1 = 0$, and $\nu_2 > 0$. On a strict open-access active-research equilibrium branch, an expansion of country 1's inference capacity increases world growth and laboratory 2's invention and frontier ownership shares. Country 1's two leadership shares both fall. For every fixed retention probability $\omega \in [0,1]$, these conclusions hold on a nonempty region of primitive parameters in which the prescribed research choices are optimal against all feasible research deviations and both governments optimally authorize commercial access.*

The corollary applies the same research mechanism with country 2 as the feedback recipient. Its unvalidated and validated intensities are $\eta\chi_2$ and $\eta\chi_2(1+\nu_2)$, the rival intensity is $\eta\chi_1$. Conditional on no innovation, an unvalidated laboratory acquires feedback with probability $f =$

[4]See more detailes in (B.1).

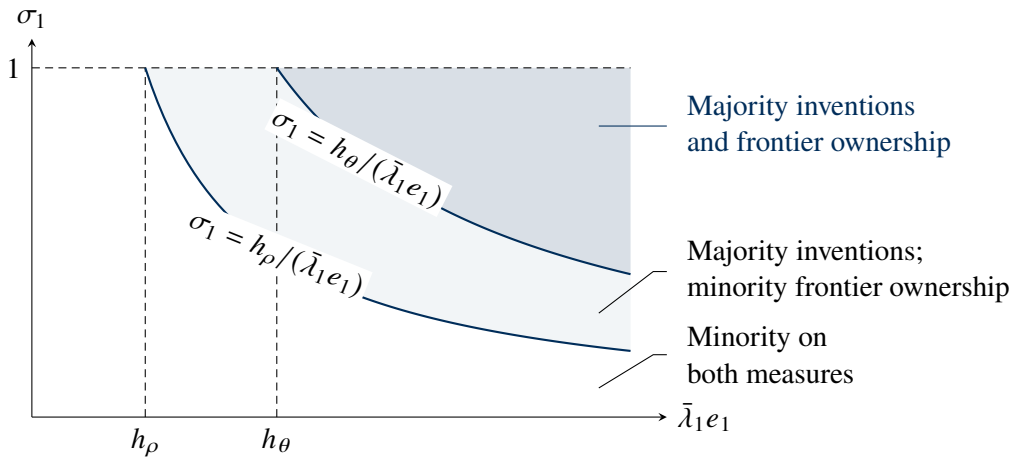


Figure 1. Deployment, feedback rights, and two majority thresholds. For positive retention and feasible thresholds, $h_j = f_j^*/(1 - f_j^*)$ satisfies $h_\theta > h_\rho$. The boundaries are the analytic conditions in (22). The horizontal axis is potentially informative exposure after training and the vertical axis is its legally usable fraction. The middle region has a majority of inventions and a minority of frontier time. The schematic spacing illustrates the ordering of the analytic boundaries.

$\Lambda_{21} e_1/(1 + \Lambda_{21} e_1)$. The same expansion can support faster global innovation while shifting commercial technology ownership abroad. Public or private delivery determines whether this deployment occurs; the allocation of usable feedback determines who obtains its direct research advantage. A financing advantage alone is therefore insufficient to establish a domestic leadership advantage.

On these equilibrium branches, expansion raises the discounted value of quality under either feedback assignment. The recipient determines the distribution of research success; the income generated by faster quality growth determines the project's investment return. The next section combines these effects with corporate claims, fiscal finance, and the government's delivery decision.

4 National investment and technological leadership

Infrastructure changes current production and future productivity, thereby changing the value of existing income claims. Corporate investors value the income claims they can acquire. A national government also values the productivity gains received by domestic workers. This section identifies that difference, derives the fiscal and timing conditions for construction, and combines them with the research equilibrium.

4.1 Income claims and the national return

Write $R_k = R_1(k)$ and $\Pi_k = \Pi_1(k)$ for domestic infrastructure receipts and ordinary-sector profits at capacity k. Fix the operating policies at each capacity, including the current actions chosen in anticipation of delivery. Let M_L, M_H be the associated quality-weighted kernels and define

$$G_k = (I - M_k)^{-1}\mathbf{1}, \qquad \Gamma = M_L(G_H - G_L). \tag{23}$$

A unilateral construction decision leaves submitted current research and access unchanged, so the construction date uses M_L and the following dates use M_H. The low-capacity value G_L continues those prescribed operating choices. For normalized income vectors $y = (y_L, y_H)$, their construction return before paying the engineering cost is

$$\mathcal{F}[y] = M_L\left[(I - M_H)^{-1}y_H - (I - M_L)^{-1}y_L\right]. \tag{24}$$

The operator is linear in income claims. A constant claim $y_L = y_H = y$ therefore gains $y\Gamma$ from the project. Existing income claims give an investor a stake in the induced quality improvement as well as a source of collateral. Appendix E derives the return for baseline and incremental corporate claims.

Let π^R_{1k} denote the domestic laboratory's net distribution and put

$$V^R_{1k} = (I - M_k)^{-1}\pi^R_{1k}, \qquad c_{1k} = L_1 + R_k + \Pi_k + \pi^R_{1k}. \tag{25}$$

The second expression is national operating consumption before construction and fiscal costs. Research labor and training inputs are already deducted in the laboratory distribution. Let $\mathcal{J}_{R\Pi}$ be the construction return to infrastructure and ordinary profits, $\mathcal{J}_C$ the return to all domestic corporate distributions, and $\mathcal{J}_G$ the national return. Then

$$\begin{aligned} \mathcal{J}_G - \mathcal{J}_{R\Pi} &= L_1\Gamma + M_L(V^R_{1H} - V^R_{1L}), \\ \mathcal{J}_G - \mathcal{J}_C &= L_1\Gamma. \end{aligned} \tag{26}$$

The first line separates labor and laboratory gains. The laboratory term can be negative when the project strengthens a foreign competitor. The second line isolates the income outside the corporate contracting set: workers benefit from the higher productivity of earlier model generations. Their future wages remain household income under the contracts specified in Section 2.4.

A corporate investor can select among the claims it is offered. Let y_k^θ be the receipts associated with an admissible assignment $\theta \in \Theta$, and define

$$B(x) = \sup_{\theta \in \Theta} \mathcal{J}[y^\theta](x), \qquad \overline{B} = \max_x B(x). \tag{27}$$

This envelope allows an organization to exclude a claim whose owner loses from the project. When $J > \overline{B}$, every admissible corporate assignment has a negative construction surplus at every state. Waiting, fairly priced borrowing, and claim exchanges then leave rejection optimal, even with unrestricted construction finance.

These expressions evaluate a specified set of operating choices. In an institutional comparison with interior research, values use the choices induced by each institution, including the construction-date research response. Section 5 implements that comparison with distinct policies during continued low capacity, construction, and operation at high capacity.

4.2 Public finance and the incentive to build

The government can assign installed infrastructure receipts and a quality-contingent tax stream to domestic investors. Let w be inherited liquid wealth, $c_G(s) \geq 0$ a current implementation cost, and $h(x) = (M_L G_H)(x)$ the construction-date value of one unit of installed receipts per unit of quality. Write $\phi(n)$ for the minimum collection cost per unit of quality needed to raise net receipts n. Assume $\phi(0) = 0$ and ϕ is increasing and convex. The enforceable net tax base is bounded by $\bar{n}$. The required constant normalized collection is

$$n^*(s) = \frac{[J + c_G(s) - w - R_L - h(x)R_H]_+}{1 + h(x)}. \tag{28}$$

If $n^* > 0$, current tax receipts and the sale of future project and tax receipts satisfy the construction budget

$$w + R_L + n^* + h(x)(R_H + n^*) = J + c_G(s). \tag{29}$$

The project is fiscally feasible when $n^*(s) \leq \bar{n}$. Convex collection costs make constant normalized receipts optimal under the given quality-contingent claims, with total implementation cost

$$C_G(s) = c_G(s) + [1 + h(x)]\phi(n^*(s)). \tag{30}$$

The tax-smoothing argument follows Barro (1979); here the endogenous quality process supplies the valuation weights. Appendix H proves the cost minimum and the sequential optimality of col-

lection after n^* becomes an inherited obligation. Higher capitalization raises project-receipt values and spreads a given fiscal requirement over a larger future income base.

The net national construction gain, relative to continuing the low operating policy, is

$$A^0 = \mathcal{J}_C + L_1\Gamma - J\mathbf{1}, \qquad A_N(s) = A^0(x) - C_G(s). \tag{31}$$

Let K_u be the quality-weighted transition under a feasible waiting financial choice u, including its successor balance sheet, and let $c_w(s,u) \geq 0$ be its additional resource cost. The government compares construction with both permanent rejection and postponement.

Proposition 2 (The national investment test)**.** *Fix the operating policies and an invariant set of financial states on which public construction is financeable. Immediate construction is optimal throughout that set if and only if*

$$A_N(s) \geq 0, \qquad A_N(s) \geq -c_w(s,u) + (K_u A_N)(s) \quad \textit{for every feasible } u. \tag{32}$$

Strict inequalities make immediate construction the unique timing policy. With a finite operating state, waiting kernel M_L, and zero waiting flow cost, these conditions are equivalent to

$$\begin{aligned} &L_1\Gamma \geq J\mathbf{1} + C_G - \mathcal{J}_C, \\ &L_1\gamma \geq (I - M_L)(J\mathbf{1} + C_G - \mathcal{J}_C), \qquad \gamma = (I - M_L)\Gamma. \end{aligned} \tag{33}$$

The first inequality requires the labor-income gain to cover the project cost remaining after corporate returns. The second values earlier completion against saving or waiting for a different research state. The quality acceleration gain satisfies

$$\gamma = M_L(M_H - M_L)G_H. \tag{34}$$

On the one-recipient active branch, an effective feedback expansion gives $\Gamma > 0$ and $\gamma > 0$. The investment test then gives a finite income threshold for immediate construction. Appendix G provides its explicit expression. It keeps the economy's income base fixed and includes engineering, fiscal, and postponement costs.

4.3 The joint institutional equilibrium

Let $W_1^P(s)$ and $W_1^G(s)$ be national discounted consumption under corporate and national investment control from the same initial capacity, frontier ownership, feedback, and liquid wealth. The follow-

ing result combines the allocation of research feedback with the income and financing conditions above.

Proposition 3 (National investment and foreign innovation gains)**.** *Suppose $\Lambda_{21} > 0$, all other feedback exposures are zero, $\nu_1 = 0$, and $\nu_2 > 0$. Country 1's project strictly increases its effective inference capacity. There is a nonempty primitive region in which the two institutional environments have stationary Markov-perfect equilibria with open commercial access and strictly optimal positive research by both incumbent and challenger laboratories, such that*

$$\begin{aligned} I^P = 0, \qquad I^G = 1, \qquad W_1^G > W_1^P, \\ g_H > g_L, \qquad \rho_{1H} < \rho_{1L}, \qquad \theta_{1H} < \theta_{1L}. \end{aligned} \tag{35}$$

The private rejection holds for every admissible corporate rights assignment, even with its funding constraint relaxed. Public construction is immediately optimal at every precompletion state in the maintained financial state set. Tax claims are fairly priced, all inherited obligations are honored, and tax collection and commercial access remain optimal after installation. The construction can require strictly positive tax backing and incur strictly positive collection losses.

For given labor endowments, sufficient conditions are the strict laboratory inequalities in Appendix C, $J > \overline{B}$, the strict net stopping tests (32) and (33), fiscal feasibility, and the foreign access bound in Appendix F.1. These conditions hold jointly for finite labor endowments and every fixed $\omega \in [0, 1]$.

Useful feedback improves the foreign laboratory's research productivity, whereas knowledge embodied in earlier model generations raises labor productivity in both countries. The same project can therefore reduce domestic technological shares and raise domestic consumption. After all domestic corporate distributions are included, the national return still contains the unassigned labor-income gain $L_1\Gamma$. Construction is preferred when that gain covers the remaining engineering and real fiscal costs and the value of postponement. Fiscal capacity makes the preferred delivery choice feasible.

Proof outline. Choose research costs satisfying (19) and a positive research bound small enough to make both laboratories' marginal incentives strict throughout their choice sets. Useful foreign feedback then gives $\Gamma, \gamma > 0$ and the leadership comparisons in Corollary 1. Corporate returns are bounded; choose $J > \overline{B}$. The private investment option is consequently zero for every admissible assignment.

The quality-contingent fiscal contract funds public construction at a bounded real collection cost. Because the labor-income benefit is $L_1\Gamma$ and the acceleration benefit is $L_1\gamma$, finite labor endowments can satisfy both strict investment inequalities. When country 1 owns the model, its com-

mercial opening leaves the useful feedback process unchanged and earns export royalties. When country 2 owns it, opening country 1 also improves the feedback available to its own laboratory. A sufficient foreign income stake makes opening optimal along the private path, the construction path, and the installed path. These comparisons include joint access, financing, and delivery deviations. The inherited payment obligations and dated market clearing complete the stationary equilibrium. Appendix F.1 gives the bounds and full proof.

Section 6 studies how fiscal transfers can implement this investment while leaving construction under private control.

Corollary 2 (Public investment with interior research)**.** *The parameter region in Proposition 3 can be chosen so that, for every sufficiently small $\tau > 0$, allowing effort above η with the convex labor requirement $\mathcal{L}_i^\tau$ in (D.1) preserves stationary Markov-perfect equilibria with all the strict comparisons in (35). Both laboratories' research choices are then interior. The tax stream and its issue price are reoptimized for the resulting operating policies, and the inherited obligation continues to be honored.*

On the smooth capacity-increase region of Corollary 3, equilibrium research at high capacity exceeds its permanent-low-capacity value for both laboratories in every ownership and feedback state: $r_{iH}^\tau(x) > r_{iL}^\tau(x)$. Country 1's invention and frontier ownership shares nevertheless fall. The interior equilibrium combines greater domestic research effort with a smaller domestic share of world research success.

The construction-period research policy anticipates next-period capacity, whereas the private nonconstruction policy anticipates continued low capacity. Appendix F.2 gives the proof. The next section evaluates these responses with quadratic research costs and bilateral feedback.

5 Research responses and the incidence of investment

The numerical illustration varies the allocation of information generated by domestic deployment while holding the infrastructure project and labor endowments fixed. Both laboratories learn from deployment in both countries, and research has the quadratic labor requirement $\mathcal{L}_i(r) = .002r + (5/2)r^2$. The foreign laboratory's baseline research productivity is twenty percent higher, domestic inference capacity expands from one to two, and the engineering cost equals three percent of the domestic labor-income flow. The assigned primitives provide a controlled, uncalibrated illustration; Appendix K gives their complete values and the equilibrium calculations.

Table 4. The incidence of investment at three feedback assignments

σ	$\Delta\rho_1$	$\Delta\theta_1$	$\Delta\bar{q}_1$	Δg	$\Delta W_1/W_1^P$
.35	−.286	.798	1.893	.207	1.728
.80	−.389	−.559	1.953	.202	1.719
.95	.201	.054	1.939	.183	1.551

The first two outcome columns are percentage-point changes. The third is the change in innovation probability in percentage points per model date; the fourth is 100 times the change in log-quality growth per date. The final column is the percentage change in lifetime national welfare. Welfare starts from domestic frontier ownership, unvalidated feedback and zero liquid wealth. Public construction uses its construction-date research policy before installed capacity becomes available.

The single varying primitive is

$$\Lambda(\sigma) = \begin{pmatrix} \sigma & .1 \\ 1-\sigma & .1 \end{pmatrix}, \qquad \sigma \in \{.01, .02, \ldots, .99\}. \tag{36}$$

The first column allocates the potential informative exposure generated in country 1 between the domestic and foreign laboratories. Its sum is fixed. The second column gives both laboratories access to information from foreign deployment. Thus $\sigma = .80$ assigns eighty percent of country 1's potential exposure to its own laboratory; the research gain also depends on validation, baseline research productivity, endogenous effort and the distribution of operating states.

5.1 Research effort, invention shares and frontier ownership

Figure 2 plots high-minus-low capacity changes in the two stationary leadership shares, in percentage points, against the domestic feedback assignment σ. Circles and squares identify assignments supporting public construction and private nonconstruction under the stated policy conditions; crosses report the remaining operating comparisons. Table 4 presents innovation, growth, and welfare outcomes for three assignments satisfying those conditions. At $\sigma = .35$, the domestic invention share falls while its frontier ownership share rises. At $\sigma = .80$, both leadership shares fall; at $\sigma = .95$, both rise. Domestic research, the domestic innovation rate, national welfare and world growth increase in all three cases.

The case $\sigma = .80$ separates the scale of domestic research from its relative technological position. Research rises in every ownership and feedback state in both countries. Domestic research increases by between .00584 and .02198, depending on the state. Its stationary innovation rate rises from .07239 to .09192 per date. Yet foreign innovation also responds, and the domestic invention share falls by .389 percentage points. The technological incidence of potential exposure depends

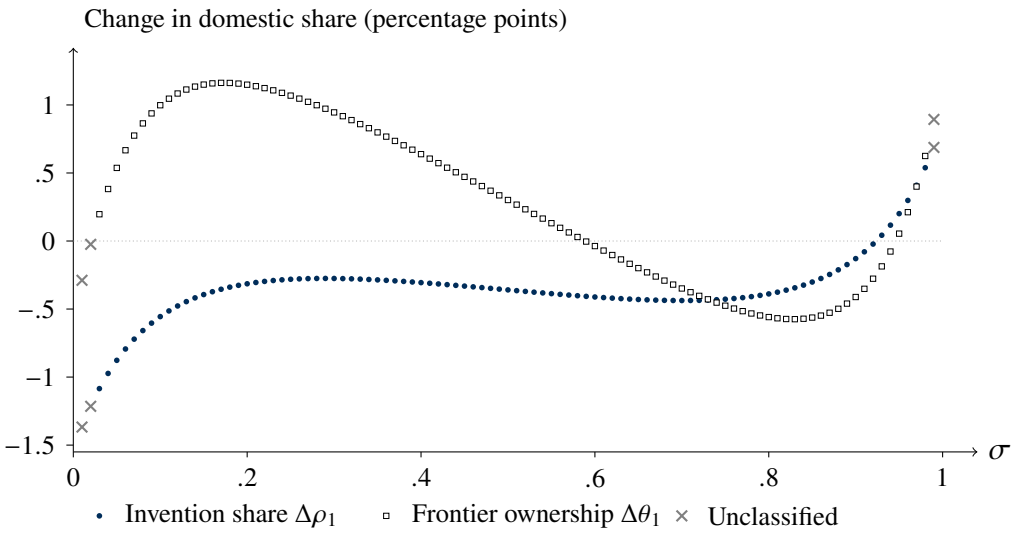


Figure 2. Leadership outcomes at all 99 sampled feedback assignments

Filled circles and open squares report the 96 sampled assignments that pass the investment and policy conditions. Gray crosses indicate $\sigma = .01, .02, .99$, where the uniform investment-timing bound falls below its required threshold. Primitives other than the feedback assignment in (36) are held fixed; see Appendix K.

on baseline research productivity, validation, research incentives and the induced distribution of operating states.

The difference between the two leadership measures has a precise accounting interpretation. Taking changes in (18) gives

$$\Delta\theta_1 = \Delta\rho_1 - \Delta\left[\frac{\mathrm{Cov}_\pi(q, I_1)}{\bar{q}}\right]. \tag{37}$$

At $\sigma = .35$, the covariance term divided by the aggregate innovation rate falls from .04930 to .03846, a decline of 1.084 percentage points. Hence the .286-point fall in invention share is consistent with an .798-point rise in the frontier ownership share. States with high innovation probabilities become less disproportionately associated with domestic frontier ownership. The innovation-weighted measure and the time-weighted measure consequently move in opposite directions.

At a given information state, an increase in the rival's effective research intensity lowers the domestic laboratory's current-period innovation probability, holding its own intensity fixed. Capacity expansion changes both laboratories' research choices and access to validated information. In the $\sigma = .80$ equilibrium, domestic innovation increases, but less rapidly than worldwide innovation, so its share falls.

Table 5. Sources of the national welfare gain

	$\sigma = .35$	$\sigma = .80$	$\sigma = .95$
Labor-income gain	202.8120	201.2903	182.8024
Corporate-income gain	12.4926	12.4692	12.3860
Engineering cost	−30.0000	−30.0000	−30.0000
Collection loss	−.0070	−.0074	−.0083
Net welfare gain	185.2976	183.7521	165.1801

Discounted present values per unit of initial quality, from the initial state used in Table 4. Each row uses the actual construction-date and continuation policies. The initially assigned tax stream remains fixed after issue; collection losses are deducted at all dates. Rounding can affect the last displayed digit of a column sum.

5.2 The sources of the welfare gain

The national return can be decomposed along the actual public and private paths. Let M_L be the quality-weighted and discounted transition matrix under permanent low capacity, M_H the installed high-capacity kernel, and M_T the construction-date kernel. Research at the construction date uses current low capacity and anticipates high capacity next date. Write $G_k = (I - M_k)^{-1}\mathbf{1}$ for $k \in \{L, H\}$ and let ΔV_1^{corp} be the difference in the present value of all domestic corporate net income across these paths. The national accounts imply

$$\Delta W_1 = \underbrace{L_1\left[\mathbf{1} + M_T G_H - G_L\right](x_0)}_{\text{labor-income gain}} + \underbrace{\Delta V_1^{\text{corp}}(x_0)}_{\text{corporate-income gain}} - J - C_G(x_0, 0). \tag{38}$$

Corporate income includes infrastructure receipts, ordinary-sector profits and laboratory profits net of research costs under each path's research policy. The last two terms are engineering expenditure and real tax-collection losses. Domestic claim payments and taxes are transfers within the national accounts. Table 5 evaluates this decomposition for the three feedback assignments.

The corporate component measures the change in total domestic corporate income along the two equilibrium paths. Private construction incentives are evaluated separately using the cash-flow envelope in (27), which holds current research fixed in the construction deviation and permits the investor to exclude claims harmed by the project. At $\sigma = .80$, the corporate-income gain is 12.47 and the engineering cost is 30, whereas the labor-income gain is 201.29. This decomposition identifies the income component underlying the national incentive to invest. Across the three cases, faster quality growth produces most of the national benefit. The corporate gain varies little, and the engineering cost is fixed. This incidence reflects the fixed background labor endowment in the illustration. It is the same income-rights distinction used by the analytical investment comparison: households receive a growth return outside the admitted corporate income claims.

Table 6. The transition at $\sigma = .80$

	Domestic frontier probability		Cumulative domestic inventions		Discounted consumption gain
Model dates	Private	Public	Private	Public	$\Delta W_1(T)$
5	.6334	.5869	.1770	.2520	−15.33
20	.4854	.4785	1.2089	1.5947	89.73
50	.4838	.4782	3.3801	4.3522	173.44

Frontier probabilities and expected invention counts are measured after T transitions. The consumption gain sums discounted consumption over dates $t = 0, \ldots, T-1$, per unit of initial quality. Engineering expenditure occurs at $t = 0$, and collection losses are charged at every date.

The policy calculations verify corporate rejection, public construction, commercial access and the financing obligations at the solved operating policies. The corporate return envelope is below the project cost even when funding is unrestricted. Public construction dominates all feasible waiting and financing choices under the stated fiscal institution. These sufficient checks hold at 96 of the 99 sampled assignments; 34 of those assignments lower both domestic leadership shares. Appendix K reports the conditions and numerical accuracy checks, including the three assignments marked separately in Figure 2.

5.3 The transition and the timing of consumption

The stationary comparison has a counterpart along the investment path. Table 6 follows the same initial state at $\sigma = .80$. After twenty model dates, the probability of domestic frontier ownership is .4785 with public construction and .4854 with private nonconstruction. Expected cumulative domestic innovations are 1.5947 and 1.2089, respectively. Expected quality relative to its initial level is 1.1992 under public construction and 1.1530 under private nonconstruction. Domestic invention activity expands even while the probability of controlling the frontier is lower.

The upfront engineering cost makes the discounted consumption gain negative over the first five dates. The gain becomes positive over the first twenty dates and reaches 173.44 over the first fifty dates, approaching the lifetime gain of 183.75. Immediate investment is optimal because its lifetime benefit exceeds the cost and the value of postponement. That comparison allows the initial expenditure to precede the consumption gains it creates.

6 Fiscal support and private implementation

The national investment gain identifies a case for fiscal support. Whether that support requires public control depends on the contracts available to the government and private investors. This section derives the return and funding requirements and the construction grants that satisfy them.

6.1 Pledgeable returns and the investment frontier

An investor can receive an income stream while being able to pledge only a fraction of it to outside financiers. At a low-capacity operating state x, let M_0 be the quality-weighted transition generated by the current research and access choices, which are held fixed in the investor's deviation. Let M_L and M_H describe the specified subsequent operating allocations at low and high capacity, and put

$$G_k = (I - M_k)^{-1}\mathbf{1}, \qquad h(x) = (M_0 G_H)(x), \qquad l(x) = (M_0 G_L)(x). \tag{39}$$

These capitalization factors value discounted future quality from the current state, incorporating transition dynamics and discounting. The current kernel M_0 incorporates the operating response to anticipated construction, while M_L describes permanent low capacity.

Consider an infrastructure investor with claims on ordinary-sector profits. Let $\lambda \in [0, 1]$ be its share of baseline profits and $\alpha \in [0, 1]$ its share of the increment $\Delta\Pi = \Pi_H - \Pi_L$. Its receipts are

$$S_L = R_L + \lambda\Pi_L, \qquad S_H = R_H + \lambda\Pi_L + \alpha\Delta\Pi. \tag{40}$$

Its post-construction distributions have current value $H(x) = h(x)S_H$. The enforceable fraction of this stream is $\kappa^p \in [0, 1]$. The organization may sell claims with total market value at most $\kappa^p H(x)$. Investors receive the contracted share of actual future distributions, with repayments contingent on the realized state. Competitive investors use the household's discount factor, so an issue of value f reduces the organization's retained continuation value by exactly f.

Let $w \geq 0$ be the organization's liquid resources before current operating receipts and after any payments to acquire its portfolio. The frontiers below are conditional on that portfolio. Construction can be funded if and only if

$$J \leq \mathcal{F}(x; \alpha, \lambda, \kappa^p, w) := w + S_L + \kappa^p h(x) S_H. \tag{41}$$

The construction return relative to permanent low capacity is

$$\mathcal{J}(x; \alpha, \lambda) := h(x)S_H - l(x)S_L. \tag{42}$$

Fair issuance supplies current resources at an equal reduction in retained continuation value. A project can have a positive net present value, $\mathcal{J} - J > 0$, and still face a funding shortfall at the construction date.

Proposition 4 (Appropriation, pledgeability and full financial implementation)**.** *For any operating continuations satisfying $\|M_k\|_\infty < 1$, the following statements hold.*

1. *The return and funding frontiers are*

$$\begin{aligned} \mathcal{J} &= hR_H - lR_L + \alpha\Delta\Pi\, h + \lambda\Pi_L(h-l), \\ \mathcal{F} &= w + R_L + \lambda\Pi_L + \kappa^p h(R_H + \lambda\Pi_L + \alpha\Delta\Pi). \end{aligned} \tag{43}$$

   Thus baseline rights affect investment incentives whenever construction changes the capitalization factor $h - l$. They increase available funding even when $h = l$.

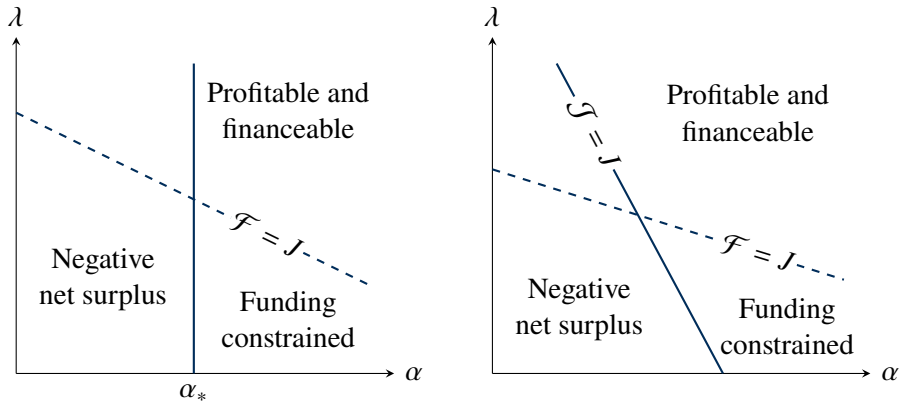
2. *If $h > l$, $\Delta\Pi > 0$ and $\Pi_L > 0$, the investment boundary $\mathcal{J} = J$ slopes downward in the (α, λ) plane. The funding boundary $\mathcal{F} = J$ slopes downward when $\kappa^p > 0$ and is horizontal when $\kappa^p = 0$. The investment boundary is strictly steeper. If they intersect, the baseline receipts at the intersection are*

$$S_L^\times = \frac{(1-\kappa^p)J - w}{1 + \kappa^p l}. \tag{44}$$

3. *With full pledgeability, $\kappa^p = 1$, any project satisfying $\mathcal{J} \geq J$ is immediately financeable. More generally, full pledgeability removes a funding obstacle to any optimal current construction decision whose no-construction continuation has nonnegative value.*

Figure 3 separates the return requirement $\mathcal{J} \geq J$ from the funding requirement $\mathcal{F} \geq J$. In panel (a), $h = l$: baseline income claims relax the funding constraint and cancel from the incremental investment return. In panel (b), $h > l$: construction raises the present value of baseline receipts, so both baseline and incremental participation increase the investment return. The solid return boundary then slopes downward and is steeper than the dashed funding boundary.

Full pledgeability makes a privately profitable project immediately financeable. Public backing can relax a funding shortfall at the same household discount rate, whereas a negative net present value, $\mathcal{J} - J < 0$, requires an additional income claim or transfer. These frontiers distinguish the two obstacles that fiscal support must address; the timing decision also compares construction with waiting.



(a) Equal capitalization $h = l$

(b) Higher capitalization under construction: $h > l$

Figure 3. Income claims and the two investment frontiers. Incremental participation α is on the horizontal axis and baseline participation λ on the vertical axis. The diagrams depict an interior crossing with $\Delta\Pi > 0$ and $0 < \kappa^p < 1$; boundaries outside the admissible unit square leave only the corresponding feasible regions. When $h = l$, baseline participation increases funding. When $h > l$, it also increases the investment return by raising the present value of baseline income. The funding and return boundaries are (43), and their slopes are (H.1). A point on the positive-surplus side of the solid boundary and above the dashed funding boundary has positive current construction surplus and sufficient funding; optimal timing also accounts for waiting.

6.2 Fiscal implementation and the role of control

Suppose construction and its completion are contractible. A government may offer a construction-contingent net grant $\mathfrak{g} \geq 0$, without acquiring control of the organization. The investor observes the current offer before choosing construction and financing. Research and access choices have already been submitted and cannot be revised; the investor does not observe them. The enforceable grant is settled at the investment date and is available to meet the current funding requirement. The offer is available only for current construction; all subsequent policies are held fixed. At a given decision node, let H be the unencumbered post-construction continuation value, and let W be the value of the best no-construction choice after subtracting the common current cash $w + S_L$. The latter includes both retained-earnings choices and any future fiscal policy already in place. Define $\Omega = H - W$, and let $\mathcal{F}$ be the funding frontier before the current grant. If the no-construction alternative is permanent low capacity, $\Omega = \mathcal{J}$.

The minimum current grant is

$$\mathfrak{g}_{\min} = \max\{0, J - \Omega, J - \mathcal{F}\}. \tag{45}$$

The return deficit $J - \Omega$ and funding deficit $J - \mathcal{F}$ determine the required transfer. A feasible grant at this level implements current construction with acceptance at indifference; a strictly larger grant makes the private incentive strict. Appendix H derives the formula.

Contractible completion allows the government to implement the desired physical investment under private control. Public control has an additional role when relevant project attributes or decision rights cannot be supplied through that contract. A voluntary public acquisition also has to compensate the existing owner for relinquishing its rights.

The importance of specifying these restrictions follows the public-ownership analyses of Hart et al. (1997) and Besley and Ghatak (2001). In those models, ownership changes investment incentives because relevant quality or investment decisions cannot be fully specified in advance. Here, the contingent-grant benchmark identifies the enforceable decision that would make a transfer sufficient.

6.3 Construction grants and the value of waiting

The one-node grant formula makes current construction weakly optimal given the actual waiting value. To account for the incentives created by a whole schedule, retain the same operating policies and financial state. Let $A_P(s)$ be the private net construction surplus without grants, and $\mathcal{F}(s)$ its resources available for current construction. A nonnegative Markov grant $\mathfrak{g}(s)$ is paid only if construction occurs now. Rejection voids the current offer. Feasible waiting choices and their kernels are unchanged by the schedule, and the enforceable borrowing base before a construction order excludes future grants. Put

$$(\mathcal{K}v)(s) = \sup_{u \in U(s)} (K_u v)(s), \qquad b(s) = \max\{0, A_P(s), A_P(s) + J - \mathcal{F}(s)\}. \tag{46}$$

Waiting has zero incremental flow after subtracting the continued-low income value, as in (I.2). Assume A_P and $\mathcal{F}$ are bounded and every waiting kernel has row mass at most $d < 1$. The recipient can reject construction forever.

Proposition 5 (The minimum Markov construction grant)**.** *There is a unique bounded solution*

$$B^* = \max\{b, \mathcal{K}B^*\}. \tag{47}$$

The pointwise smallest nonnegative grant schedule that makes immediate construction feasible and optimal at every state is

$$\mathfrak{g}^*(s) = B^*(s) - A_P(s). \tag{48}$$

The grant $\mathfrak{g}^* + \epsilon$, for any constant $\epsilon > 0$, makes immediate construction strictly optimal. The private continuation value under $\mathfrak{g}^*$ is B^*, so anticipated future grants are included in the implementation condition.

Suppose the recipient constructs at indifference, higher grants weakly raise fiscal resource costs without changing the physical allocation or the operating policies, and the government's resulting net gain from immediate construction $A_N^{\mathfrak{g}^*}$ satisfies the national investment test, including every feasible waiting transition. If the government can fund $\mathfrak{g}^*$ at every state and honor the implied obligations, this schedule is also a sequentially optimal government grant rule, conditional on those operating policies and subsequent fiscal obligations.

The difference $B^* - b$ is the additional rent needed to offset waiting for future offers. The value B^* is the smallest continuation value that covers the funding requirement and makes immediate construction optimal. The government implements the schedule when it can finance the grants and prefers financing immediate construction to postponing it. This characterization connects infrastructure support to inverse optimal stopping (Kruse and Strack, 2015, 2019).

A two-state specialization shows why financing support creates a dynamic rent. Suppose waiting leads to state 2 with quality-weighted factors $m_{12}, m_{22} \in (0, 1)$, so

$$K = \begin{pmatrix} 0 & m_{12} \\ 0 & m_{22} \end{pmatrix}.$$

Let the private construction deficits be $d_1, d_2 > 0$, so $A_P = (-d_1, -d_2)'$. Current funds cover the engineering cost in state 1. In state 2 the funding gap satisfies $d_2 < h \leq J$. Then

$$b = \begin{pmatrix} 0 \\ h - d_2 \end{pmatrix}, \qquad B^* = \begin{pmatrix} m_{12}(h - d_2) \\ h - d_2 \end{pmatrix}, \qquad \mathfrak{g}^* = \begin{pmatrix} d_1 + m_{12}(h - d_2) \\ h \end{pmatrix}. \tag{49}$$

A grant of h is needed to finance construction in state 2, leaving the recipient a rent $h - d_2$. That future rent raises the minimum grant in state 1 above its current profitability deficit even though state 1 has no financing gap. The premium compensates for the option to obtain future funding support. A future funding constraint therefore raises the current implementation cost even at a state with sufficient liquid resources.

7 Conclusion

AI infrastructure connects local deployment, research information, and the income generated by technological progress. The model identifies equilibria in which corporate investors reject a project and tax-backed public construction raises domestic welfare and world growth while reducing the investing country's invention and frontier ownership shares. Useful feedback strengthens foreign

research, while better models raise domestic labor income. With local feedback rights, deployment can instead overcome a domestic disadvantage in baseline research productivity.

The quadratic-cost illustration shows that domestic research effort and innovation can increase while domestic leadership shares fall. The two shares can also move in opposite directions. Along the transition, upfront construction costs precede the productivity gains that make investment desirable.

Fiscal support can operate through public construction or private implementation. Pledgeable receipts determine funding, appropriable income determines investment incentives, and anticipated grants determine the value of waiting. When construction is contractible, a contingent grant can implement investment under private control, subject to fiscal feasibility and the government's continuation incentives.

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Online Appendix

A Operating markets, national accounts, and recursive verification

All prices, income flows, and financial positions in this appendix are measured per unit of current quality. The accounts apply to corporate income assignments and to the public revenue and tax claims used in the main analysis.

A.1 Competitive production and trade

The ordinary producer solves $\max_{m\geq 0}\{b_i\sqrt{m}-u_i m\}$, giving $u_i = b_i/(2\sqrt{m_i})$ at a positive allocation. An application obtains p_i per inference after paying the authorized frontier royalty, or by using the predecessor. Positive inference therefore gives $u_i+v_i = p_i$. If compute is slack, $v_i = 0$ and $m_i = m_i^*$. If compute binds, subtracting the physical constraints gives $m_i = E_i - K_i$. Complementarity yields (4) and $e_i = \bar{e}_i - \varepsilon_T r_i$. The maintained active region makes inference and background employment positive at every feasible research choice, fixing the normalized wage at one.

Infrastructure and ordinary fixed-factor receipts satisfy

$$R_i + \Pi_i = b_i\sqrt{m_i} + p_i(e_i + \varepsilon_T r_i). \tag{A.1}$$

This follows from $u_i + v_i = p_i$, market clearing, and zero rents on unused capacity. Adding net laboratory distributions gives

$$L_i + R_i + \Pi_i + \pi_i^R = L_i - \mathcal{L}_i(r_i) + b_i\sqrt{m_i} + p_i e_i + P_i.$$

Thus laboratory input expenditures enter its private payoff, while their national opportunity cost appears as reduced inference and background production. Their sum is the total resource cost shown in national consumption.

Gross domestic output is

$$y_i = L_i - \mathcal{L}_i(r_i) + b_i\sqrt{m_i} + (p_i + o_i\mu D_i)e_i.$$

Outgoing royalties are $\mathbf{1}_{\{\ell\neq i\}} o_i\mu D_i e_i$, and incoming royalties are $\mathbf{1}_{\{\ell=i\}}\sum_{j\neq i} o_j\mu D_j e_j$. Final-good shipments settle these payments because assets are held domestically. Subtracting outgoing and

adding incoming royalties, then deducting construction and collection resources, gives (11). Summing across countries gives (12).

A.2 Resident budgets and enforceable claims

Let S_i be the operating receipts assigned to the investment controller, and let z_i denote its distribution or public rebate. The remaining resident operating income is $Y_i^{\text{out}} = L_i + R_i + \Pi_i + \pi_i^R - S_i$. Write $\mathfrak{p}_i$ for current payments from the controller to residents on inherited financial contracts, and $\mathfrak{a}_i$ for current proceeds received from residents under new financial contracts. Both are signed: saving redemption and shareholder cash calls enter with their actual direction. Let T_i be gross domestic collection and $n_i = T_i - \Phi_i$ its net receipts. Let $K_i^{\text{build}} = (J + c_G)I$ in country 1 and zero in country 2. The two accounts are

$$\begin{aligned} c_i &= Y_i^{\text{out}} + z_i + \mathfrak{p}_i - \mathfrak{a}_i - T_i, \\ z_i &= S_i - \mathfrak{p}_i - K_i^{\text{build}} + \mathfrak{a}_i + n_i. \end{aligned} \tag{A.2}$$

Their sum is $c_i = L_i + R_i + \Pi_i + \pi_i^R - K_i^{\text{build}} - \Phi_i$. Domestic acquisition payments, retained deposits, and claim servicing cancel in the same way. A negative laboratory distribution is a funding obligation carried by its financial claim; transferring that claim carries its agreed funding obligation to the buyer.

For a promised stream $H_t Q_t$, beginning next date, competitive issue proceeds at current quality Q_0 equal $\mathbb{E}\sum_{t\geq 1}\beta^t(Q_t/Q_0)H_t$. If $|H_t| \leq \overline{H}$, the absolute price is bounded by $\overline{H}\beta a/(1-\beta a)$. The claim's payment rule is recorded in the inherited state. In particular, the tax obligation n^* issued on construction remains that obligation after feedback or ownership changes. A hidden unilateral operating deviation holds existing payments and issue proceeds fixed; the contracted price continues to apply. Safe retained deposits instead earn the household gross return $1/\beta$. Their redemption is a domestic household liability.

The financial bounds can be satisfied simultaneously with positive consumption. Let $\overline{\mathcal{F}}_i$ bound gross resident payments under all admitted financial actions, including purchases of claims, deposit redemption, and specified shareholder funding obligations other than the laboratory's direct research costs. Such a finite bound follows from bounded balance sheets, bounded payment rates, and the valuation bound above. Let $\overline{T}_i$ and $\overline{\Phi}_i$ bound gross collection and its real cost, and let $\overline{\mathcal{L}}_i = \max_{r\in[0,\bar{r}_i]}\mathcal{L}_i(r)$. A sufficient primitive resource and resident-funding restriction is

$$L_i > \overline{\mathcal{L}}_i + p_i\varepsilon_T\bar{r}_i + \overline{\mathcal{F}}_i + \overline{T}_i + J_i + \overline{\Phi}_i. \tag{A.3}$$

Here $\bar{r}_i$ is the applicable research bound, $J_1 = J + \sup_s c_G(s)$ in the sufficient resource bound, and $J_2 = 0$. The restriction is conservative: wage income alone covers the gross calls, and na-

tional consumption remains strictly positive after the largest engineering and collection resource uses. Ordinary profits and nonnegative model royalties only relax these bounds. The main existence construction chooses finite labor endowments after fixing the bounded financial and research primitives.

A.3 Recursive verification

The full state records liquid assets, pledged income shares, and inherited tax obligations in addition to capacity, ownership, and feedback. Financial positions need not form a finite set. On the maintained bounded state set, normalized feasible rewards and values are bounded, and the quality-weighted transition has row mass at most $\beta a < 1$. For fixed opponents, the Bellman operator is therefore a contraction whenever its feasible sets are nonempty. The investment option formulation subtracts liquid wealth and the appropriate no-construction income value; the retained-earnings problem is proved in Appendix I. Its scalar nonnegative-receipt formulation applies to the infrastructure and ordinary-profit portfolios specified there. Exclusion of construction for the broader corporate envelope uses the bounded signed income values and the relaxed-finance stopping problem in Appendix E.

To verify a proposed strategy against history-dependent deviations, iterate its Bellman inequality along any unilateral deviation through date T. The equilibrium value bounds the accumulated payoff plus discounted continuation. Since $Q_T/Q_0 \le a^T$ and normalized values are bounded, the terminal term tends to zero at rate $(\beta a)^T$. This establishes optimality over admissible unilateral histories once all current joint controls and inherited-state decisions have been checked. It also justifies the series $(I - M)^{-1} = \sum_{t\ge 0} M^t$. Contraction verifies a player's best response; the joint equilibrium additionally requires the laboratory, authorization, construction, and fiscal inequalities.

Long-run outcomes use physical transition probabilities. On a recurrent ergodic operating class, the time average of the innovation probability converges to its stationary expectation. The bounded martingale difference between realized innovations and these probabilities has vanishing time average, so $t^{-1}\log(Q_t/Q_0) \to \bar{q}\log a$ almost surely. If normalized consumption c_{it} is bounded above and bounded away from zero, consumption in levels, $C_{it} = Q_t c_{it}$, has the same long-run log-growth rate as quality:

$$\frac{1}{T}\log\frac{C_{iT}}{C_{i0}} = \frac{1}{T}\log\frac{Q_T}{Q_0} + \frac{1}{T}\log\frac{c_{iT}}{c_{i0}} \longrightarrow \bar{q}\log a.$$

State and transition rewards give frontier ownership shares and invention shares, respectively.

Table 7. The marginal allocation of infrastructure returns

Binding input	Added input	dR_i	$d\Pi_i$	$d\bar{e}_i$
Compute: $E_i - K_i > m_i^*$	Electricity	$u_i/2$	$u_i/2$	0
	Compute	$p_i - u_i/2$	$-u_i/2$	1
Power: $E_i - K_i < m_i^*$	Electricity	p_i	0	1
	Compute	0	0	0

Each entry is the derivative of normalized infrastructure receipts R_i, ordinary-sector profit Π_i, or pretraining inference capacity $\bar{e}_i$ with respect to one unit of the indicated input, holding the other input and research fixed. At the boundary, allocation is continuous and the appropriate one-sided derivatives apply. Training subtracts $\varepsilon_T r_i$ from inference in either region.

B Infrastructure composition and the deployment bottleneck

Electricity and compute are complementary in AI use, but they have different outside uses. This distinction determines which investments expand the model market and which domestic incomes gain from expansion. Inference capacity is given by (5). An increase in infrastructure spending raises deployment only when it relaxes the binding constraint. The composition of the project therefore enters both the research problem and the institutional comparison.

Proposition 6 (Capacity composition and the distribution of returns)**.** *In the active operating region, away from $E_i - K_i = m_i^*$, marginal changes in infrastructure receipts, ordinary-sector profits, and inference capacity are given by Table 7. When compute binds, additional power raises ordinary production but does not raise inference. Additional compute raises inference and reduces ordinary-sector profit. When power binds, additional compute has no operating value, whereas additional power raises inference. A deployment target $\bar{e}_i > z$ requires both*

$$K_i > z, \qquad E_i > m_i^* + z. \tag{B.1}$$

Proof. When compute binds, $m_i = E_i - K_i$, $\Pi_i = b_i\sqrt{m_i}/2$, and $R_i = \Pi_i + p_i K_i$. Differentiation gives the first two rows. When power binds, $m_i = m_i^*$, $u_i = p_i$, $v_i = 0$, and $R_i = p_i E_i$; this gives the last two rows. Equation (B.1) follows directly from the minimum in (5). □

Figure 4 displays the deployment restrictions in the (K_i, E_i) plane. The diagonal separates compute scarcity from power scarcity, and the shaded region supports inference above z before training. Reaching that region requires both $K_i > z$ and $E_i > m_i^* + z$. At fixed research, expansion of the slack input alone leaves deployment unchanged. The figure therefore identifies the complementary capacity needed for infrastructure investment to affect learning. In the leadership comparison

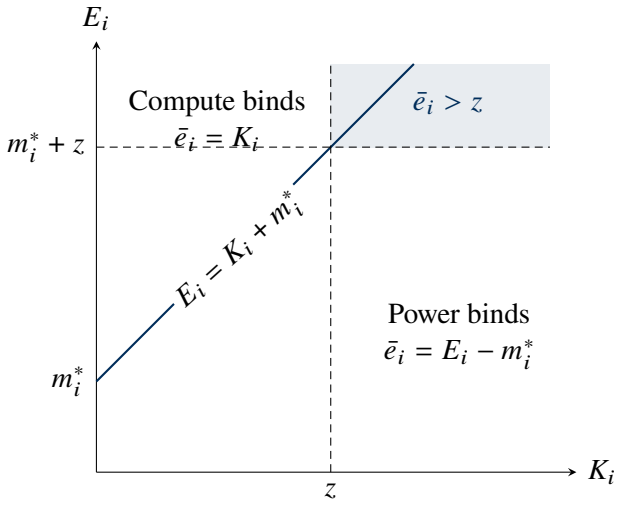

Figure 4. Electricity, compute, and a deployment target. The boundaries are (5) and (B.1); the diagram displays the analytical feasibility regions.

in Section 3, z is an endogenous threshold determined by the competing laboratories' innovation technologies.

The distinction also matters for spillovers. A joint power-and-compute expansion can raise both R_i and Π_i. When computing capacity binds, a compute expansion with fixed power displaces ordinary production. For example, with research held at zero, the increase in country 1's current income after installation is

$$\Delta c_1 = \Delta R_1 + \Delta \Pi_1 + \mathbf{1}_{\{\ell=1\}} \mu D_1 \Delta \bar{e}_1. \tag{B.2}$$

If the foreign laboratory owns the frontier, the additional model royalty accrues abroad. With a compute-only project, the national current-income return can then be smaller than the infrastructure company's return because $\Delta \Pi_1 < 0$. The national investment test includes this displacement of ordinary production alongside the induced quality gain and fiscal implementation cost.

C Incumbent innovation and retained feedback

Use the research intensities x, y, z and validation probability f defined in (20), and set $\delta = \delta_1$. These intensities apply in both incumbent and challenger roles. Define

$$\begin{aligned} &A = x + z, && B = y + z, && C_0 = B + \delta, \quad C = \delta + (1 - \omega)B, \\ &D = AC + fB, && Z = (1 + A)C + f(1 + B), \\ &D_0 = AC_0 + fB, && Z_0 = (1 + A)C_0 + f(1 + B). \end{aligned} \tag{C.1}$$

The invariant distribution gives

$$\rho_1 = \frac{xC + fy}{D}, \qquad \theta_1 = \frac{(xC + fy)Z_0 - fy\omega(y - x)}{ZD_0}, \qquad g = \frac{D}{Z}\log a. \tag{C.2}$$

Consequently,

$$\rho_1 - \theta_1 = \frac{\omega f z C(y - x)^2}{ZD_0 D}. \tag{C.3}$$

C.1 A strict active-research laboratory equilibrium

Fix open access and a capacity plan that is permanent L, permanent H, or current L followed by H. At every full operating state let both laboratories choose η. Innovation retains each existing feedback set with probability ω, independently conditional on the current state. The bounds below are uniform over $\omega \in [0, 1]$.

Put $\bar{t}_i = \chi_i(1 + \nu_i)$, $H_* = \bar{t}_1 + \bar{t}_2$, $\bar{q} = \eta H_*/(1 + \eta H_*)$, and $d_\eta = \beta[1 + (a - 1)\bar{q}] < 1$. Let

$$\begin{aligned} K_i^r &= c_i^r + \kappa_1 + \kappa_2, \\ E_i^r(\eta) &= \frac{\eta K_i^r + \beta\bar{q}(a + 1)v_H}{1 - d_\eta}, \\ K_i^D &= 2\beta\{a\bar{t}_i + \varepsilon_T\lambda_i\}. \end{aligned} \tag{C.4}$$

A finite-parameter sufficient condition for the active-research policies to be strict laboratory best responses is

$$\frac{\beta(a - 1)\chi_i v_L}{(1 + \eta H_*)^2} - c_i^r - \kappa_i - K_i^D E_i^r(\eta) > 0, \qquad i = 1, 2. \tag{C.5}$$

This condition is stated directly in primitives and permits positive incumbent effort. It converges to the strict inequalities (19) as η tends to zero.

To prove it, let U_i be the zero-research trial value: it is zero for a challenger, and v_L, v_H, or $\mathcal{B}_L + \beta v_H$ for an incumbent in the respective capacity environment. The successor incumbent value v lies in $[v_L, v_H]$. For the proposed positive-research policies, current net laboratory income differs from its trial reward by at most ηK_i^r in absolute value. No-innovation changes of feedback do not affect U_i. On an innovation, the reference continuation can change by at most $(a + 1)v_H$. The fixed-policy discounted operator therefore has a two-sided residual at most $\eta K_i^r + \beta\bar{q}(a + 1)v_H$ at U_i. Its value V_i satisfies $\|V_i - U_i\|_\infty \le E_i^r$.

For every current deviation $r_i \in [0, \eta]$, the sum of absolute derivatives of the three race probabilities is at most $2\bar{t}_i$. The sum of absolute derivatives of the conditional feedback law is at most $2\varepsilon_T\lambda_i$. Retention after innovation is independent of current effort, conditional on the outcome and

inherited feedback. Consequently the absolute row-sum derivative of the quality-weighted kernel is at most K_i^D. Replacing V_i by U_i in a current deviation derivative has error at most $K_i^D E_i^r$.

With $h = t_i(d_i)r_i$ and $H = t_j(d_j)\eta$, the exact derivatives at the trial continuation are

$$-c_i^r - \kappa_i + \frac{\beta v t_i(d_i)\{a(1+H)-1\}}{(1+h+H)^2} \quad \text{for an incumbent,}$$

and

$$-c_i^r + \frac{\beta a v t_i(d_i)(1+H)}{(1+h+H)^2} \quad \text{for a challenger.}$$

The incumbent derivative is bounded below by the left side of (C.5). The challenger derivative has the same lower bound plus a strictly positive term. Thus each objective is strictly increasing throughout its continuous action interval. Every one-date deviation is excluded, at off-path feedback states as well as on the equilibrium path. Discounted verification establishes the proposed laboratory equilibrium. Because the margins are strict, small changes in research productivity, physical capacities, costs, and feedback parameters preserve the branch.

The following primitive restrictions establish nonemptiness. Fix $1 < a < 1/\beta$, positive capacities, and baseline research productivities. Choose $\varepsilon_T > 0$ sufficiently small that $(p_i + \mu D_i)\varepsilon_T < \beta(a-1)\chi_i v_L$ for both countries. Choose a strictly positive labor slope below the remaining gap. This gives (19). A sufficiently small positive η then satisfies (C.5) and the physical active inequalities. The construction-date argument uses v_H as the successor value and is covered by the same lower bound.

C.2 Verification of research incentives at finite intensity

The bound in (C.5) is sufficient but can be conservative. Research at finite intensity can instead be checked directly from the exact finite-state values. Consider a one-recipient feedback benchmark and fix the rival's proposed research at η. For a laboratory and state, let t be its research productivity in that feedback state, H its rival's effective intensity, and $L = 1 + H$. Let W_O and W_R be its continuation values conditional on its own and its rival's innovation, averaging the retained-feedback draws. Write the no-innovation continuation as

$$W_0(r) = S - \frac{\Delta}{b - dr}, \qquad b - dr > 0.$$

If the laboratory's training reduces unvalidated useful feedback, $b = 1 + \lambda\bar{e}$, $d = \lambda\varepsilon_T$, and Δ is the difference between the appropriate validated and unvalidated continuation values. If the deviation does not change useful feedback, take $d = \Delta = 0$, $b = 1$, and $S = W_0$. This representation applies

both when the information goes to the deviator and when it goes to its rival; it uses the deviator's value at the resulting feedback states.

Let $C = c_i^r + \mathbf{1}_{\{\ell=i\}}\kappa_i$ be the current marginal cost including displaced own royalties. Apart from a constant, the one-date payoff is

$$-Cr + \beta \frac{W_0(r) + atrW_O + aHW_R}{L + tr}.$$

Define $K = atLW_O - taHW_R - tS$. Its derivative has the sign of the polynomial

$$P(r) = \{\beta K - C(L + tr)^2\}(b - dr)^2 + \beta\Delta(tb - dL - 2tdr). \tag{C.6}$$

The positive denominator is $(L + tr)^2(b - dr)^2$. Equation (C.6) follows by differentiation, without a small-intensity approximation.

Write $P(r) = \sum_{j=0}^{n} a_j r^j$, $n \le 4$. Its Bernstein coefficients on $[0, \eta]$ are

$$B_k = \sum_{j=0}^{k} a_j \eta^j \frac{\binom{k}{j}}{\binom{n}{j}}, \qquad k = 0, \ldots, n. \tag{C.7}$$

Since $P(\eta u) = \sum_{k=0}^{n} B_k \binom{n}{k} u^k (1-u)^{n-k}$, strictly positive B_k imply $P(r) > 0$ throughout the entire feasible interval. Applying this test at every laboratory and feedback/ownership state verifies strict upper-bound research. One may subdivide the interval and apply the same test after an affine change of variable; the Bernstein bound applies throughout each subinterval. Together with the exact discounted value system, this gives sufficient conditions for the research branch at finite intensity. Governments' access, construction, and financing choices retain their separate optimality conditions.

C.3 Stationary outcomes with arbitrary retention

For $\nu_2 = 0$, foreign validation does not affect research intensities, inference, or continuation payoffs on the active-research branch. The relevant state is (ℓ, d_1). Set $d = d_1$ and use the notation in (C.1). At $d = 0$, the probabilities of no innovation, home innovation, and foreign innovation are respectively $1/(1 + A)$, $x/(1 + A)$, and $z/(1 + A)$. At $d = 1$ the corresponding probabilities are $1/(1 + B)$, $y/(1 + B)$, and $z/(1 + B)$.

The feedback marginal is independent of the frontier owner. Its transition probabilities from zero to one and from one to zero are

$$\frac{f}{1 + A}, \qquad \frac{\delta + (1 - \omega)B}{1 + B} = \frac{C}{1 + B}. \tag{C.8}$$

The stationary feedback probabilities are therefore

$$m_0 = \frac{(1+A)C}{Z}, \qquad m_1 = \frac{f(1+B)}{Z}. \tag{C.9}$$

Average home and total innovation probabilities are $(xC + fy)/Z$ and D/Z. Their ratio gives ρ_1, while $g = (D/Z)\log a$.

To derive the frontier ownership share from the invariant distribution, let u_d be the stationary probability that home owns the frontier and feedback is d. Define the substochastic no-innovation matrix and the innovation-retention matrix by

$$T = \begin{pmatrix} (1-f)/(1+A) & f/(1+A) \\ \delta/(1+B) & (1-\delta)/(1+B) \end{pmatrix}, \qquad R_\omega = \begin{pmatrix} 1 & 0 \\ 1-\omega & \omega \end{pmatrix}.$$

Stationary owner masses solve

$$(u_0, u_1) = (u_0, u_1)T + (m_0, m_1)\,\mathrm{diag}\left(\frac{x}{1+A}, \frac{y}{1+B}\right) R_\omega. \tag{C.10}$$

The inverse $N = (I - T)^{-1}$ exists because both states have a strictly positive innovation probability. Its expected absorption-time vector is

$$(N\mathbf{1})_0 = \frac{Z_0}{D_0}, \qquad (N\mathbf{1})_1 - (N\mathbf{1})_0 = -\frac{y-x}{D_0}. \tag{C.11}$$

Multiplying (C.10) by $\mathbf{1}$ now yields the ownership-share expression in (C.2). All four joint states communicate, and each has a self-loop, so the joint chain is irreducible and aperiodic. These stationary expressions are therefore the limiting outcomes from every initial ownership and feedback state.

The same absorption system gives the probability that home produces the next innovation, conditional on the current feedback state:

$$p_0^{\text{next}} = \frac{xC_0 + fy}{D_0}, \qquad p_1^{\text{next}} = \frac{y(A+f) + \delta x}{D_0}. \tag{C.12}$$

They are independent of the current owner because efforts do not depend on ownership on this branch. They are independent of ω because retention acts after the next innovation. Their derivatives with respect to f are $z(y-x)C_0/D_0^2$ and $\delta z(y-x)/D_0^2$, both strictly positive. Thus capacity can change the nationality of the next innovation from the same initial ownership and feedback state, in addition to changing long-run shares. This is the next realized innovation, which may fol-

low several dates without innovation. At the current fixed feedback state and upper-bound research, next-period innovation probabilities themselves are unchanged by capacity.

Straight subtraction gives (C.3). The exact derivatives with respect to the home validation probability are

$$\begin{aligned} \frac{\partial \rho_1}{\partial f} &= \frac{z(y-x)C}{D^2} > 0, \\ \frac{\partial (g/\log a)}{\partial f} &= \frac{(y-x)C}{Z^2} > 0, \\ \frac{\partial \theta_1}{\partial f} &= \frac{z(y-x)C\{Z_0^2 - \omega(y-x)(1+A)C_0\}}{Z^2 D_0^2} > 0. \end{aligned} \tag{C.13}$$

For the last sign, $Z_0 \geq (1+A)C_0$ and $(1+A)C_0 > y - x \geq \omega(y-x)$. Thus the bracket is strictly positive. The strict research optimality conditions make these derivatives equilibrium comparative statics on the specified primitive region.

Suppose $x < z < y$. Rearranging $2(xC + fy) > D$ gives (21). Write

$$\begin{aligned} a_2 &= (y-z)(1+B) > 0, \\ a_1 &= 2xC(1+B) + 2y(1+A)C_0 - 2y\omega(y-x) - (1+A)CB - (1+B)AC_0, \\ a_0 &= (x-z)(1+A)CC_0 < 0. \end{aligned}$$

Then $\theta_1 > 1/2$ is equivalent to $a_2 f^2 + a_1 f + a_0 > 0$, and its unique positive boundary is

$$f_\theta^* = \frac{-a_1 + \sqrt{a_1^2 - 4a_2 a_0}}{2a_2}. \tag{C.14}$$

At the invention boundary, positive retention implies $\theta_1 < \rho_1 = 1/2$, proving $f_\theta^* > f_\rho^*$. To see that the two-majority region is nonempty at feasible probabilities, fix $\chi_1 < \chi_2 < \chi_1(1+\nu_1)$ and choose $\delta > 0$ so that $\delta(\chi_2 - \chi_1)/\{\chi_1(1+\nu_1) - \chi_2\}$ lies strictly between zero and one. At any fixed $\omega > 0$, both thresholds converge to this number as $\eta \downarrow 0$. Their strict ordering then gives a nonempty feasible range with $\rho_1 > 1/2 > \theta_1$ at every sufficiently small positive η. Choosing costs as above supports the same research branch. This gives a nonempty region with different invention and frontier ownership majorities.

For $\omega = 0$, each innovation starts the next cycle at $d = 0$. The duration and winner of the new cycle are independent of the previous winner because research intensities do not depend on ownership. Writing $p = \theta_1 = \rho_1$, (C.11) implies that the mean cycle duration is $\tau = 1 + (1-p)/z$. Home can win successive cycles. Expected uninterrupted home and foreign ownership durations obey $\mathcal{T}_1 = \tau + p\mathcal{T}_1$ and $\mathcal{T}_2 = \tau + (1-p)\mathcal{T}_2$. These equations give (C.18) and its comparative statics.

C.4 Quality growth and the institutional comparison

Let G_d be the quality value under installed capacity, and define $q_0 = A/(1+A)$ and $q_1 = B/(1+B)$. It is independent of the current frontier owner. Subtracting its two Bellman equations gives

$$G_1 - G_0 = \frac{\beta(a-1)(q_1 - q_0)G_0}{1 - \beta\{(1-q_1)(1-\delta) + aq_1\omega - (1-q_0)f\}} > 0. \tag{C.15}$$

The denominator is positive because $\beta\{(1-q_1)(1-\delta) + aq_1\omega\} \leq \beta(1 - q_1 + aq_1) < 1$.

Suppose construction increases f from f_L to f_H and leaves the strictly optimal research efforts unchanged. Then $R = (M_H - M_L)G_H$ is strictly positive in the unvalidated home states and zero in the validated states. The resolvent identity gives $G_H - G_L = (I - M_L)^{-1}R$. From either validation state, the next state is unvalidated with positive probability: an unvalidated state can remain so, and a validated state depreciates with positive probability conditional on no innovation. Hence

$$\Gamma = M_L(G_H - G_L) > 0, \qquad \gamma = M_L R > 0$$

in every ownership and feedback state. The growth conditions in Proposition 9 therefore hold for all $\omega \in [0, 1]$.

At a low-capacity date followed by high capacity, the continuation is G_H. The quality value of a low date followed by construction is $\widehat{G}_L = \mathbf{1} + M_L G_H$. Since $G_H = \mathbf{1} + M_H G_H$,

$$\widehat{G}_{L1} - \widehat{G}_{L0} = \{1 + \beta(1-q_0)(f_H - f_L)\}(G_{H1} - G_{H0}) > 0. \tag{C.16}$$

Thus validated information also raises the quality value during the construction transition. Appendix F.1 uses these quality differences to verify authorization and investment together with enforceable fiscal funding.

C.5 Positive foreign learning and the full operating state

Retain both feedback indicators in the full state (ℓ, d_1, d_2). At the local-learning benchmark only Λ_{11} is positive. The unused indicator d_2 cannot be newly validated and, from an inherited value of one, becomes zero almost surely: conditional no-innovation depreciation has probability bounded away from zero. The states with $d_2 = 0$ form the unique recurrent class. Within that class, positive research, positive inference, and $0 < \delta_1 < 1$ make the ownership-and-home-feedback process irreducible and aperiodic. The invariant eigenvalue one of the full finite transition matrix is therefore simple, although the full matrix is reducible at the benchmark. Its invariant distribution and discounted values are continuous and locally differentiable functions of the transition entries. This

follows from their finite linear systems, or from the inverses of $I - P + \mathbf{1}\pi$ and $I - M$, respectively. The same conclusion holds with the two country labels reversed at the foreign-learning benchmark.

The finite research margins in (C.5) are uniform over feedback and ownership states. They remain positive under sufficiently small changes in the second laboratory's learning productivity and usable-feedback entitlements, so both laboratories continue to choose η in every state. Continuity also preserves the strict derivatives of leadership and growth with respect to home inference, and strict comparisons between specified low and high capacities. With useful feedback reaching both laboratories, outcomes are obtained from the full eight-state invariant distribution. If the second feedback-rights row remains zero, its indicator is transient and the benchmark closed forms continue to describe the recurrent class.

For the institutional comparison, keep the fiscal state set compact and the verified finance, stopping, and access inequalities uniformly strict, including the installed policies with inherited tax obligations. Current issue-price schedules and previously contracted payment streams are held fixed in an unobserved current-period unilateral policy deviation; equilibrium schedules are then recalculated for the perturbed primitives. The same continuity argument preserves that verified candidate. This establishes a local continuation of the verified equilibrium.

C.6 Proof of Corollary 1

The research-optimality argument extends to the feedback matrix in (9). A deviation in laboratory i's effort changes inference in country i by $-\varepsilon_T$ per unit effort. For each recipient j, the absolute derivative of its validation probability is at most $\varepsilon_T \Lambda_{ji}$ because $0 \le w_i(o_i) \le 1$. Differentiating the product distribution of the two conditional feedback draws gives an absolute row-sum bound $2\varepsilon_T \sum_j \Lambda_{ji}$. The race-probability derivative is unchanged. Replace the derivative constant in (C.4) by

$$K_i^D(\Lambda) = 2\beta \left(a\bar{t}_i + \varepsilon_T \sum_j \Lambda_{ji} \right). \tag{C.17}$$

The two-sided fixed-policy residual E_i^r is unchanged: the reference continuation is independent of feedback. Thus (C.5), with the new derivative constant, again verifies every incumbent and challenger deviation throughout $[0, \eta]$. In particular, own training's effect on a foreign laboratory's usable feedback is included in the optimality check. For every finite matrix Λ, the strict primitive cost condition and sufficiently small positive η satisfy this bound.

When only Λ_{21} is positive and $\nu_1 = 0$, useful information belongs to laboratory 2. Under open access, both countries' inference is independent of frontier ownership on the active-research branch.

Relabel the learning laboratory as the first laboratory in the proof of Proposition 1. The relevant physical transition has exactly the same form, with

$$x = \eta\chi_2, \quad y = \eta\chi_2(1+\nu_2), \quad z = \eta\chi_1, \quad f = \frac{\Lambda_{21}(\bar{e}_1 - \varepsilon_T\eta)}{1+\Lambda_{21}(\bar{e}_1 - \varepsilon_T\eta)}.$$

Its derivatives in (C.13) are strictly positive. Since $df/d\bar{e}_1 > 0$, country 2's invention and frontier ownership shares rise, world growth rises, and country 1's complementary shares fall.

The open-access policies can be verified. When country 1 owns the frontier, its domestic application always has access. Denying country 2 a commercial license changes no useful feedback, since applications in country 2 have zero entries in Λ. It only loses current export royalties. When country 2 owns the frontier, authorizing use in country 1 generates its current royalties and weakly improves its own useful feedback. The national continuation gain equals its domestic growth-sensitive income base times the strictly positive quality difference in (C.15), plus a bounded laboratory-value difference. For a sufficiently large finite income base, opening is optimal in every state; if access does not change feedback, the current royalty alone makes it strictly optimal. The same comparisons apply at low and high capacity and during anticipated construction. Research payoffs do not depend on these labor endowments once resident funding is adequate. Hence the research and access conditions hold together on a nonempty strict parameter region.

If $\nu_1 > 0$ but its entire feedback-rights row is zero, unvalidated home feedback cannot become validated. Any inherited validated home set becomes unvalidated in finite time almost surely, because conditional no-innovation depreciation has a uniformly positive probability. The unique recurrent operating class then has $d_1 = 0$, and the same long-run formulas apply conditional on a verified open-access active-research branch. To obtain a joint equilibrium extension, take both $\nu_1 > 0$ and home feedback rights sufficiently small. Continuity of the finite-state invariant distribution preserves the strict comparative statics, while continuity of all strict research and government-access margins preserves optimality, including at inherited $d_1 = 1$ states. The continuation holds in a neighborhood of the specified feedback assignment.

C.7 Leadership spells under complete reset

Under complete reset, the expected length of a continuous home leadership spell, allowing the incumbent to replace its own model, is

$$\mathcal{T}_1 = \frac{1}{1-\theta_1} + \frac{1}{z}, \qquad \mathcal{T}_2 = \frac{1+z}{z\theta_1} - \frac{1}{z}. \tag{C.18}$$

Improved home feedback both lengthens home leadership spells and shortens foreign leadership spells. Incumbents can use deployment knowledge to retain frontier ownership through subsequent innovations.

D Interior research near a strict active equilibrium

D.1 Interior research and the incidence of deployment gains

The preceding closed forms use research at its upper bound. To examine whether their economic comparisons require that constraint to bind, we allow research above η at an increasing marginal labor requirement. Fix $\bar{r}_i > \eta$ and replace the linear labor requirement by

$$\mathcal{L}_i^{\tau}(r) = \ell_i^r r + \psi_{\tau, M_i^c}(r - \eta), \qquad r \in [0, \bar{r}_i], \tag{D.1}$$

where $\tau > 0$, $M_i^c < \infty$, and

$$\psi_{\tau,M}(t) = \begin{cases} 0, & t \le 0, \\ t^2/(2\tau), & 0 < t \le \tau M, \\ Mt - \tau M^2/2, & t > \tau M. \end{cases} \tag{D.2}$$

This is a continuously differentiable convex labor requirement. Its additional marginal cost is bounded by M, so resource requirements remain bounded even for deviations far above the benchmark effort bound. Choose $\bar{r}_i - \eta$ small enough to preserve positive inference, background employment, and resident funding for every feasible action. The additional labor is an actual production cost and enters national income as well as laboratory profits.

Let ξ denote an inference-capacity or feedback-rights parameter. At a strict active-research equilibrium of the original economy, let $V_i^0(\xi)$ be the laboratory's value, and define its marginal gain at the benchmark effort bound by

$$\mathfrak{m}_i(x;\xi) = \frac{\partial}{\partial r_i}\left\{\pi_i^{R,0}(x, r_i, \eta; \xi) + M_{x\cdot}(r_i, \eta; \xi) V_i^0(\xi)\right\}\Big|_{r_i = \eta} > 0. \tag{D.3}$$

Here $\pi_i^{R,0}$ uses the original linear research cost, and M is the quality-weighted continuation kernel. The derivative includes the effect of training on every laboratory's usable application feedback.

Proposition 7 (A local interior research equilibrium)**.** *Fix an open-access active-research operating equilibrium whose research objectives have uniformly positive derivatives throughout* $[0, \eta]$.

Suppose each government's open-access choice has a strict positive advantage, and the active production and resident-funding inequalities have strict slack. For sufficiently large finite M_i^c, sufficiently small positive $\bar{r}_i - \eta$, and every sufficiently small $\tau > 0$, the economy with (D.1) has a stationary operating equilibrium in which each laboratory's action is its unique global best response in $[0, \bar{r}_i]$, and

$$r_i^\tau(x;\xi) = \eta + \tau \mathfrak{m}_i(x;\xi) + O(\tau^2) \in (\eta, \bar{r}_i). \tag{D.4}$$

The equilibrium is locally unique near this branch. Its research choices, discounted values, and transition probabilities converge in C^1 to their original-branch counterparts as $\tau \downarrow 0$. When the physical operating chain has a unique recurrent class, invention shares, frontier ownership shares, and growth also converge in C^1. Consequently all strict capacity comparisons in Proposition 1 and Corollary 1 persist for sufficiently small τ. The result also applies to a low-capacity date followed by construction. In that case the research and access inequalities must hold with the corresponding construction-date continuations.

This extension permits research to respond to infrastructure through continuation profits and usable feedback. In particular,

$$\frac{\partial r_i^\tau(x;\xi)}{\partial \xi} = \tau \frac{\partial \mathfrak{m}_i(x;\xi)}{\partial \xi} + O(\tau^2). \tag{D.5}$$

Its sign is determined by the capacity derivative of the marginal research return, $\partial_\xi \mathfrak{m}_i$. The following result identifies a region in which both laboratories increase their research while the domestic leadership shares fall.

Corollary 3 (Rising domestic research and falling domestic leadership)**.** *Consider the feedback allocation of Corollary 1 and a smooth increase in country 1's inference capacity that raises $\mathcal{B} = \mu \sum_j D_j \bar{e}_j$, holding baseline research productivities and the unit training-input price fixed. On a nonempty strict parameter region, choose sufficiently small positive η and then sufficiently small positive τ. Equilibrium research by both laboratories rises with this capacity in every ownership and feedback state. Nevertheless, country 1's invention and frontier ownership shares fall, while world quality growth rises. With local home feedback rights instead, the same local interior extension preserves the positive home-leadership effects of Proposition 1.*

The allocation of useful feedback changes the distribution of innovation even when domestic research responds positively to investment. The corollary characterizes a neighborhood with steep marginal research costs near the former bound. Appendix F.2 verifies the corresponding welfare and delivery comparison, using the strict financing, construction, and authorization margins in Proposition 3.

Domestic invention counts. The one-recipient analytical benchmark gives an exact quantity comparison. Label the feedback recipient's intensities x, y and the other laboratory's intensity z, as in (20). With $\bar{q} = \bar{q}_1 + \bar{q}_2$,

$$\bar{q}_{\text{nonrecipient}} = z(1 - \bar{q}) = \frac{z(C + f)}{Z}, \qquad \frac{\partial \bar{q}_{\text{nonrecipient}}}{\partial f} = -\frac{zC(y - x)}{Z^2} < 0. \tag{D.6}$$

The common denominator of the single-winner contest gives this quantity effect at a fixed intensity of the nonrecipient's research. At fixed positive η, the strict quantity effect persists in the sufficiently small interior-cost neighborhood of Proposition 7, even though domestic research increases. With larger research adjustments, a lower share can instead accompany more domestic inventions: $d\bar{q}_i = \bar{q}\, d\rho_i + \rho_i\, d\bar{q}$. The three comparisons must then be evaluated separately.

D.2 Construction of the equilibrium and all continuous deviations

We prove Proposition 7 on the full finite operating state space, including both feedback indicators. Write π_i^0 for the original laboratory reward and $M(r;\xi)$ for its quality-weighted transition matrix when the research profile is r. Each row of M has sum at most $\beta a < 1$. Work in a compact neighborhood of the specified primitives in which the operating formulas are smooth and the original positive marginal returns have a common lower bound $b_i > 0$. Condition (C.5), with (C.17) when applicable, supplies such a bound. The argument also applies if the exact deviation derivatives verify this bound without using that sufficient estimate.

First fix a preliminary action interval $[0, \eta + h_0]$ on which the original linear-cost active-production inequalities retain slack. Derivatives of the baseline payoff and kernel are bounded on this compact interval. Let $\overline{V}_i$ bound the original equilibrium values in the primitive neighborhood, and choose a finite number M_i^c such that

$$M_i^c > \sup_{x,r,\xi,\, \|V\|_\infty \le \overline{V}_i + 1} \left| \partial_{r_i} \pi_i^0(x, r; \xi) + \partial_{r_i} M_{x\cdot}(r;\xi) V \right| + 1. \tag{D.7}$$

The supremum is independent of the additional research labor cost. Shrink $0 < h_i \le h_0$ so that the additional labor, which never exceeds $M_i^c h_i$, preserves all the original resource and resident-funding inequalities. Put $\bar{r}_i = \eta + h_i$. This ordering avoids an unbounded labor requirement for large deviations when τ is small.

For a vector $z = (z_i(x))_{i,x}$ with $0 < z_i(x) < M_i^c$, set $r_i(x) = \eta + \tau z_i(x)$. On this part of the cost schedule, the additional research labor requirement is $\tau z_i(x)^2/2$. The proposed-policy value is therefore

$$V_i(z, \tau, \xi) = [I - M(\eta + \tau z; \xi)]^{-1} \left[\pi_i^0(\eta + \tau z; \xi) - \frac{\tau}{2} z_i^2 \right], \tag{D.8}$$

where squaring is componentwise. The inverse exists by $\beta a < 1$. The one-date first-order conditions, holding continuation strategies fixed, become

$$F_{ix}(z, \tau, \xi) := z_i(x) - \partial_{r_i} \pi_i^0(x, \eta + \tau z; \xi) - \partial_{r_i} M_{x\cdot}(\eta + \tau z; \xi) V_i(z, \tau, \xi) = 0. \tag{D.9}$$

At $\tau = 0$, the value in (D.8) is $V_i^0(\xi)$ and does not depend on z. Hence $F = z - \mathfrak{m}(\xi)$ and $D_z F = I$. The implicit-function theorem gives a unique smooth local solution $z(\tau, \xi)$ with $z(0, \xi) = \mathfrak{m}(\xi)$. The strict bounds $0 < \mathfrak{m}_i < M_i^c$ imply $0 < z_i(\tau, \xi) < M_i^c$ for small τ. Taking $\tau M_i^c < h_i$ puts every proposed action strictly inside the new feasible interval. Smoothness of the primitives implies (D.4) and (D.5), uniformly on a smaller compact neighborhood. No derivative of a continuation strategy enters the current best-response condition: continuation is held fixed in a one-date deviation.

Global optimality follows from the marginal-payoff bounds. Fix the candidate continuation V_i^τ and the candidate rival action. Let $D_i^\tau(r)$ denote the derivative of the corresponding baseline one-date payoff. Its convergence to the original derivative is uniform for $r \in [0, \eta]$, so $D_i^\tau(r) > b_i/2$ there. Thus no action below the former bound can be optimal. On $[\eta, \eta + \tau M_i^c]$, the derivative of the payoff net of the additional research cost is

$$D_i^\tau(r) - (r - \eta)/\tau.$$

There is a common finite bound on $|\partial_r D_i^\tau(r)|$ because the continuation values remain within $\overline{V}_i + 1$. For small τ the second derivative is consequently strictly negative on this interval. Its first derivative is positive at the left endpoint and negative at the right endpoint, the latter by (D.7). It has exactly one zero, which is the action given by (D.9). Finally, on $[\eta + \tau M_i^c, \bar{r}_i]$, the payoff derivative is $D_i^\tau(r) - M_i^c < 0$ by the same bound. This excludes every feasible continuous action outside the candidate. Bounded rewards and $\beta a < 1$ give discounted verification against arbitrary multistage laboratory deviations.

The proposition gives uniqueness in a neighborhood of this branch. The argument extends to current low capacity followed by high capacity by adjoining the construction-date states to the finite system; their successor block is the high-capacity block. It also covers a finite collection of such capacity plans, since one can choose common strict bounds and then a common sufficiently small τ.

To verify access as well as laboratory choices, evaluate national current rewards and continuation values at the constructed research profile. The additional labor cost at that profile is $O(\tau)$. Under the candidate access rule, national discounted values therefore converge uniformly to their original values. Current rewards and the two authorization-contingent transition rows are continuous in the research profile. Every original strictly positive national gain from authorization remains

positive. This checks each government's current access deviation while the simultaneously chosen research actions and continuation strategies are held fixed. The same finite-state argument applies to the prescribed construction-date continuations. Changes in the construction decision itself require the institutional verification below.

D.3 Comparative statics and the allocation of feedback

Let P^τ be the physical probability matrix, $q_i^\tau(x)$ its country-specific innovation probability, and ϖ^τ its invariant distribution. The IFT construction gives C^1 convergence of P^τ and q_i^τ. If the eigenvalue one of P^0 is simple, the invariant distribution is a smooth function of its entries, obtained from the stationarity equations together with $\varpi\mathbf{1} = 1$. This includes the single-recurrent-class cross-feedback benchmark even if unused feedback states are transient. It follows that

$$\theta_i^\tau = \sum_{x:\ell=i} \varpi^\tau(x), \qquad \rho_i^\tau = \frac{\sum_x \varpi^\tau(x) q_i^\tau(x)}{\sum_x \varpi^\tau(x)[q_1^\tau(x) + q_2^\tau(x)]}, \qquad g^\tau = \log(a) \sum_x \varpi^\tau(x)[q_1^\tau(x)+q_2^\tau(x)] \tag{D.10}$$

converge in C^1. The denominator is strictly positive because the fixed benchmark has $\eta > 0$. Thus the strictly signed derivatives in (C.13), and their cross-feedback counterparts, persist for small τ. Strict comparisons between two specified capacities, including a majority reversal, also persist. At a simple majority threshold, the implicit-function theorem gives a nearby threshold in the modified economy. The closed-form threshold values themselves need not remain unchanged.

We next prove the strict effort response in Corollary 3. Take a smooth capacity parameter ξ that raises $\mathcal{B}(\xi)$, without changing c_i^r, κ_i, or $t_i(d_i) = \chi_i(1 + \nu_i d_i)$. For this step vary the former bound η as well. At $\eta = 0$, a laboratory that owns the frontier has value $v = \mathcal{B}/(1-\beta)$, and the other laboratory has value zero. No change in feedback can affect these trial values. Direct differentiation of the race therefore gives

$$\mathfrak{m}_i(x;\xi)\big|_{\eta=0} = \begin{cases} -c_i^r - \kappa_i + \beta(a-1)t_i(d_i)v, & \ell = i, \\ -c_i^r + \beta a t_i(d_i)v, & \ell \neq i. \end{cases} \tag{D.11}$$

Discounted laboratory values remain smooth at $\eta = 0$ even though the physical ownership chain becomes reducible: the inverse $[I - M]^{-1}$ remains bounded by $(1-\beta a)^{-1}$. Hence, uniformly over operating states and a compact capacity interval,

$$\partial_\xi \mathfrak{m}_i(x;\xi) = \frac{\beta t_i(d_i)\mathcal{B}'(\xi)}{1-\beta} \begin{cases} a-1, & \ell = i, \\ a, & \ell \neq i \end{cases} + O(\eta) > 0 \tag{D.12}$$

for sufficiently small positive η, provided $\mathcal{B}'$ is bounded away from zero. Fix such an η first. Equation (D.5) then implies a strictly positive capacity response of every research choice for sufficiently small positive τ. The leadership and growth derivatives remain those of the original strict branch up to a vanishing perturbation, proving the corollary. The construction fixes a positive η before choosing τ, preserving a uniquely recurrent operating process. The effort response converges to zero with τ at that fixed research bound.

The same proof accounts for training that reduces a foreign laboratory's feedback. The derivative estimate (C.17) and the compact bounds in (D.7) differentiate every entry of the feedback-recipient matrix. Neither the IFT nor the deviation argument treats another laboratory's validation probability as fixed when it depends on the deviator's inference capacity.

D.4 Institutional continuation

Appendix F.2 applies the operating construction to the strict institutional equilibrium in Proposition 3. It recomputes construction-date research, claim prices, and the constant normalized tax obligation, then verifies the government against all waiting, financing, and authorization deviations on the compact financial state set. That proof establishes Corollary 2 and preserves the actual-path welfare comparison.

E Income rights and the national return to growth

E.1 Construction returns and the option to wait

All vectors are indexed by the finite operating state x. For $k = L, H$, let M_k be a nonnegative matrix with row sums at most $d < 1$. For a normalized flow y_k, its prescribed-policy value is $U_k^y = (I - M_k)^{-1} y_k$. The inverse exists and is nonnegative because $(I - M_k)^{-1} = \sum_{t \geq 0} M_k^t$.

Before construction, current flow is y_L. Immediate construction costs J and gives value $y_L - J\mathbf{1} + M_L U_H^y$. Perpetual rejection gives $U_L^y = y_L + M_L U_L^y$. Subtraction gives

$$A_y = \mathcal{F}[y] - J\mathbf{1}. \tag{E.1}$$

This comparison holds current operating actions fixed, as required for a unilateral investment deviation. The future operating rules are the specified Markov continuations.

With unrestricted finance and no cost of waiting beyond discounting, the construction-option value satisfies

$$W = \max\{0, A_y, M_L W\}. \tag{E.2}$$

The maximum is componentwise. This operator is a contraction with modulus d. If $A_y < 0$ everywhere, $W = 0$ is its unique fixed point, and immediate construction is strictly suboptimal in every state. Financial restrictions only remove available construction and waiting choices; fair financial transfers do not add a construction surplus. Never constructing remains feasible, so this rejection conclusion continues to hold under such restrictions.

If $A_y > 0$ and $A_y > M_L A_y$ everywhere, $W = A_y$ is the unique fixed point. Construction is uniquely optimal. These observations establish why a positive gain relative to perpetual rejection alone does not prove immediate construction: the second inequality rules out waiting.

For a controller with financial state b, let K_u include its successor financial position under a feasible waiting action u. When the operating policy is independent of b and of the financial choice, $K_u\Gamma = M_L\Gamma$. This equality carries the acceleration comparison to the augmented operating and financial state.

E.2 Baseline and incremental income claims

Consider the receipts

$$S_L = R_L + \lambda\Pi_L, \qquad S_H = R_H + \lambda\Pi_L + \alpha\Delta\Pi, \qquad \alpha, \lambda \in [0, 1]. \tag{E.3}$$

The parameter α assigns incremental ordinary-sector profits to the controller, while λ assigns baseline profits. Define $\mathcal{J}_R = M_L(R_H G_H - R_L G_L)$.

Proposition 8 (Existing claims and the incentive to build)**.** *At the specified operating policies, the controller's construction-return vector is*

$$\mathcal{J}(\alpha, \lambda) = \mathcal{J}_R + \alpha\Delta\Pi M_L G_H + \lambda\Pi_L\Gamma. \tag{E.4}$$

Consequently, baseline participation changes the construction incentive by $\Pi_L\Gamma$, in addition to its effect on funding. At any state where $\Delta\Pi > 0$, the boundary between a positive and a negative current construction surplus is

$$\alpha_*(x, \lambda) = \frac{J - \mathcal{J}_R(x) - \lambda\Pi_L\Gamma(x)}{\Delta\Pi(M_L G_H)(x)}. \tag{E.5}$$

It slopes downward in λ when $\Gamma(x) > 0$, is horizontal when $\Gamma(x) = 0$, and slopes upward when $\Gamma(x) < 0$.

The distinction between baseline and incremental rights depends on the technology. If construction leaves the quality process unchanged, baseline income accrues along the same path with and

without the project. It cancels from the return comparison and matters only through finance. When infrastructure changes innovation, the same existing assets earn income along a different quality path. Their owners then gain or lose from the project even if their receipts per unit of quality are unchanged.

Equation (E.5) and the funding boundary characterize different restrictions on private delivery. A claim can relax both. The return boundary is evaluated at the operating policies associated with the economy under consideration. A change in rights that changes research or authorization also changes M_L and M_H; the resulting institutional comparison uses the newly determined matrices.

E.3 Proof of Proposition 8

Linearity of the value operator gives

$$U_L^S = (R_L + \lambda\Pi_L)G_L, \qquad U_H^S = (R_H + \lambda\Pi_L + \alpha\Delta\Pi)G_H.$$

Apply M_L to their difference. The infrastructure component is $\mathcal{J}_R$, the incremental-profit component is $\alpha\Delta\Pi M_L G_H$, and the baseline-profit component is $\lambda\Pi_L M_L(G_H - G_L)$. This proves (E.4). Row sums of M_L are at least $\beta > 0$, and $G_H \geq \mathbf{1}$, so $(M_L G_H)(x) > 0$. Solving $\mathcal{J}(\alpha, \lambda) = J$ proves (E.5), including the sign of its slope.

These derivatives hold the operating matrices fixed. When a change of claims changes the equilibrium operating policy, the total difference also includes the change of those matrices. No global monotonicity across distinct research equilibria is required for the identity.

E.4 National accounts and attainable claims

Adding the domestic household and issuer budgets cancels debt service, new borrowing, and domestic financial transfers. Before construction and fiscal resource costs, the result is $c_{1k} = L_1 + R_k + \Pi_k + \pi_{1k}^R$. Therefore

$$\begin{aligned} U_k^G &= (L_1 + R_k + \Pi_k)G_k + V_{1k}^R, \\ U_k^{R\Pi} &= (R_k + \Pi_k)G_k, \\ U_k^C &= (R_k + \Pi_k)G_k + V_{1k}^R. \end{aligned}$$

Applying M_L to the high-minus-low differences yields (26). The value V_{1k}^R includes challenger research costs, royalty income, and all subsequent ownership changes.

The envelope in (27) is essential when claim gains have different signs. For instance, if laboratory expansion value is negative, a controller holding infrastructure and ordinary profits can value

construction more highly than a controller also holding the laboratory. A negative surplus for the full corporate pool alone would not exclude a profitable investment by the smaller coalition. Taking the supremum over the admitted claim assignments excludes this case.

Proposition 9 (The limit of corporate aggregation)**.** *Fix the operating policies and the available corporate claims. Suppose* $\Gamma(x) > 0$ *and* $\gamma(x) > 0$ *for every operating state. Choose a construction cost* $J > \overline{B}$*. Suppose public construction is financeable at every precompletion state under consideration, its total implementation resource cost, including collection and any additional public delivery cost, is bounded by* $\overline{D}$ *in present construction-date units. Waiting has nonnegative additional resource cost. There is a finite* L_* *such that, for* $L_1 > L_*$*:*

1. *every private controller with a rights assignment in* Θ *strictly rejects construction at every state, even when its funding constraint is relaxed;*
2. *a national controller strictly chooses immediate construction at every state under consideration, after accounting for its option to wait and its fiscal resource cost.*

The comparison persists under sufficiently small perturbations that preserve the strict inequalities. If the prescribed operating policies satisfy the laboratory and authorization optimality inequalities along these institutional continuations, including joint investment and access deviations, the investment choices are part of stationary Markov-perfect equilibria.

E.5 Proof of Proposition 9

Let $\mathcal{J}_C$ be the return of all domestic corporate claims. Since Θ is compact and the value map is continuous in the claim coefficients, $\overline{B} < \infty$. If $J > \overline{B}$, every admitted private controller has $A_\theta(x) < 0$ at every state. The rejection argument in (E.2) proves the private conclusion even without financing restrictions.

Let $D(s) \in [0, \overline{D}]$ be the minimal implementation resource cost of immediate public construction at precompletion state $s = (x, b)$, apart from J. It includes current delivery costs and subsequent collection costs implied by the financing arrangement, all expressed in present current-quality units. Put

$$A_0(s) = \mathcal{J}_C(x) - J - D(s), \qquad A_G(s) = L_1\Gamma(x) + A_0(s), \qquad K = \max_x |\mathcal{J}_C(x) - J| + \overline{D}. \tag{E.6}$$

The option calculation presumes that these installed continuation costs are fully included in D and that no further independent investment opportunity follows installation. Thus $|A_0(s)| \le K$. Define

$$\underline{\Gamma} = \min_x \Gamma(x) > 0, \qquad \underline{\gamma} = \min_x \gamma(x) > 0.$$

For any feasible waiting choice u,

$$A_G(s) - (K_u A_G)(s) \geq L_1 \underline{\gamma} - (1+d)K.$$

A nonnegative current resource cost of waiting only raises the advantage of immediate construction. Also $A_G(s) \geq L_1 \underline{\Gamma} - K$. Consequently the finite bound

$$L_* = \max\left\{\frac{K}{\underline{\Gamma}}, \frac{(1+d)K}{\underline{\gamma}}\right\} \tag{E.7}$$

suffices. For $L_1 > L_*$, construction is strictly better than rejection and every feasible waiting choice evaluated at A_G. The public option Bellman operator therefore has fixed point A_G, with immediate construction uniquely optimal. This proof also allows mixtures over waiting financial choices and finance states that have different construction costs within the bound.

The result uses a common set of operating policies. To verify a joint stationary equilibrium, compute laboratory values and check their complete continuous-action optimality inequalities along the resulting private and public paths. Check government authorization after every allowed current construction choice as well as along the equilibrium path; this also excludes a simultaneous investment-and-access deviation. The investment inequalities proved above then establish the remaining Bellman maximizations. Strict inequalities and continuity give an open neighborhood of any verified candidate. The result applies to the verified operating equilibrium.

F Proof of the joint institutional comparison

F.1 Proof of Proposition 3

The proof constructs a common primitive region and verifies research, commercial access, investment, and fiscal decisions on that region.

Research and the quality process. Fix $1 < a < 1/\beta$, finite baseline research productivities, positive physical capacities with $\bar{e}_1^H > \bar{e}_1^L > 0$, a finite $\Lambda_{21} > 0$, $\nu_2 > 0$, $\nu_1 = 0$, and $0 < \delta_2 < 1$. Fix any $\omega \in [0,1]$. Choose strictly positive training and linear research-labor costs satisfying (19). This is possible because the right side is strictly positive before training is introduced. Choose a sufficiently small positive η so that the active physical inequalities and (C.5) hold, using $K_i^D(\Lambda)$ from (C.17). The latter bound includes training's displacement of inference used by the foreign laboratory. It verifies every effort in $[0,\eta]$, for incumbents and challengers, at permanent low and high capacity

and at a low-capacity construction date followed by high capacity. Thus both laboratories strictly choose η throughout the candidate equilibria.

The relevant feedback indicator is d_2; d_1 does not affect research productivity. Relabel the learning laboratory in Proposition 1 by setting

$$x = \eta\chi_2, \quad y = \eta\chi_2(1+\nu_2), \quad z = \eta\chi_1, \quad f_k = \frac{\Lambda_{21}(\bar{e}_1^k - \varepsilon_T\eta)}{1+\Lambda_{21}(\bar{e}_1^k - \varepsilon_T\eta)}.$$

Then $x, z > 0$, $y > x$, and $0 < f_L < f_H < 1$. The strict derivatives in (C.13) imply

$$g_H > g_L, \qquad \rho_{2H} > \rho_{2L}, \qquad \theta_{2H} > \theta_{2L}.$$

The two national shares sum to one, giving the stated home declines. Positive effort, depreciation, and validation make the relevant four-state ownership-and-feedback chain irreducible and aperiodic. These long-run comparisons therefore apply from every initial operating state.

Write M_L, M_H for the resulting quality-weighted kernels. Relabeling (C.15) gives $G_{k,1} > G_{k,0}$, where the second index now denotes d_2. Thus $R = (M_H - M_L)G_H$ is positive in unvalidated states and zero in validated states. The resolvent identity and positive probability of reaching an unvalidated state from either feedback state give

$$\Gamma = M_L(G_H - G_L) > 0, \qquad \gamma = M_L R > 0. \tag{F.1}$$

These inequalities concern the common quality process shared by both countries.

Commercial access. When country 1 owns the frontier, restricting country 2's license changes no useful feedback: the second column of Λ is zero and country 1 retains domestic access. Current research is already submitted. The royalty contract also leaves input allocation unchanged. Restriction therefore changes neither the innovation law nor the public project's receipts and assigned tax obligations, and strictly loses current export royalties $\mu D_2 e_2 > 0$. In a private organization holding passive laboratory claims, the lost receipts can alter retained wealth. They cannot improve its real continuation: the corporate rejection result below holds at every wealth state, while fair financial transfers cancel from national value. This dominance holds for every public construction choice, liquid balance and inherited tax obligation. It rules out a profitable joint construction, financing and access deviation by country 1.

When country 2 owns the frontier, opening country 1 raises current royalties and weakly raises laboratory 2's validation probability. A finite, explicit bound suffices for its continuation incen-

tive. Let $\mathcal{P} = \{L, H, T\}$ contain permanent low capacity, permanent high capacity, and a low date followed by high capacity. For $p = T$, define

$$G_T = \mathbf{1} + M_L G_H, \qquad V^R_{2T} = \pi^R_{2L} + M_L V^R_{2H}.$$

At frontier ownership $\ell = 2$, let

$$\Delta G_p = G_p(2,1) - G_p(2,0), \qquad \Delta V_p = V^R_{2p}(2,1) - V^R_{2p}(2,0).$$

The permanent-capacity quality differences are strictly positive. Equation (C.16), with indices relabeled, also gives $\Delta G_T > 0$. Let

$$\underline{\Delta}_G = \min_{p \in \mathcal{P}} \Delta G_p > 0, \qquad \overline{\Delta}_V = \max_{p \in \mathcal{P}} |\Delta V_p|, \qquad L_2 + R_2 + \Pi_2 > \frac{\overline{\Delta}_V}{\underline{\Delta}_G}. \tag{F.2}$$

The laboratory quantities are finite and independent of L_1, L_2. This bound makes the foreign national continuation difference positive under every listed capacity plan. If country 2 is unvalidated, the access gain at current inference e_1 is

$$\mu D_1 e_1 + \frac{\beta \Delta f_2}{1 + x + z}\big\{(L_2 + R_2 + \Pi_2)\Delta G_p + \Delta V_p\big\} > 0.$$

If $\Delta f_2 = 0$, or if feedback is already validated, current export royalties alone make openness strict. Country 2 has no obligation to service country 1's fiscal claims. Its income comparison therefore does not subtract the latter's collection costs. This verifies access along the private path, the public construction path, and after a postponed construction decision. In particular, the choice of a large domestic growth stake below does not introduce a conflicting access incentive.

Corporate rejection and actual fiscal funding. The corporate receipt vectors and their finite values are determined by the operating primitives and do not depend on the two labor endowments. Compactness of Θ implies $\overline{B} < \infty$. Choose $J > \overline{B}$. Every admitted corporate construction obstacle is then negative at every state. The stopping argument in Appendix E implies perpetual rejection even with relaxed finance. Fair borrowing, deposits and exchanges of the admitted claims cannot change that negative project surplus.

For the public regime, use a compact precompletion wealth set $w \in [0, \bar{w}]$ and its feasible waiting transitions. There are no previously issued project claims at low capacity; claims are issued on construction and remain recorded after installation. Put

$$\begin{aligned} h(x) &= (M_L G_H)(x), \\ n^*(x,w) &= \frac{[J + c_G(x,w) - w - R_L - h(x)R_H]_+}{1 + h(x)}, \\ C_G(x,w) &= c_G(x,w) + [1 + h(x)]\phi\big(n^*(x,w)\big), \end{aligned} \tag{F.3}$$

where $c_G \geq 0$ is a bounded current construction implementation cost independent of access. It may be zero. Choose a finite assigned tax cap $\bar{n} > \sup_{x,w} n^*(x,w)$. This is possible, since $n^*(x,w) \leq J + \sup_s c_G(s)$ for nonnegative w, R_L, R_H. Any fixed continuous increasing convex ϕ is bounded on this finite interval. Hence C_G has a finite uniform bound $\overline{C}$ independent of the labor endowments.

When $n^* > 0$, current tax receipts n^* and the sale of enforceable future receipts $(R_H + n^*)Q_t$ at price $h(x)(R_H + n^*)$ satisfy

$$w + R_L + n^* + h(x)(R_H + n^*) = J + c_G(x,w).$$

When $n^* = 0$, a sufficient share of future project receipts covers any remaining cash requirement. The stopping obstacle subtracts the current engineering and implementation costs once each. The formulation with $c_G = 0$ already establishes the proposition with strictly positive real collection costs. To require actual tax backing from an initial wealth w_0, choose the same cost to satisfy additionally

$$J > w_0 + R_L + \max_x h(x)R_H. \tag{F.4}$$

Together with $J > \overline{B}$, this is only a finite lower bound on J and does not alter the research inequalities. Choosing $\phi(n) > 0$ for $n > 0$ then gives a strictly positive collection loss.

Issue prices use equilibrium operating choices. A hidden unilateral research or access deviation does not retroactively change the issue price or the promised stream. The promised n^* remains an inherited state as feedback is updated. At any such history, the remaining tax obligation has value $n^* G_H(x_t)$. Conditional on honoring that obligation, the dated Jensen argument in Proposition 11 makes continued collection $n_t = n^*$ optimal; refinancing does not reduce its real cost. Country 1's access dominance holds for every inherited n^*, and country 2 does not bear these losses. Thus the operating kernel used to price the claims remains optimal after issue. With adequate labor endowments, the net taxes, collection goods and all permitted household financial calls are feasible at every history. The bound $\beta a < 1$ and bounded obligations also imply the relevant no-Ponzi conditions.

Continually reoptimized construction. Define, on the augmented state $s = (x, w)$,

$$A_N(s) = \mathcal{J}_C(x) + L_1\Gamma(x) - J - C_G(s). \tag{F.5}$$

For each feasible waiting financial choice u, let K_u include the successor wealth and any inherited financial obligation; let $c_w(s, u) \geq 0$ be its real flow cost after domestic transfers cancel. The research and access policies are independent of these financial positions. Consequently $K_u\Gamma = M_L\Gamma$ and the row masses of K_u are at most $d = \beta a < 1$.

Let

$$K = \max_x |\mathcal{J}_C(x) - J| + \overline{C}, \qquad L_*^{\text{inv}} = \max\left\{\frac{K}{\min_x \Gamma(x)}, \frac{(1+d)K}{\min_x \gamma(x)}\right\}.$$

Choose finite $L_1 > L_*^{\text{inv}}$, also large enough for positive background employment and financial feasibility. Then, at every precompletion state,

$$A_N(s) > 0, \qquad A_N(s) > -c_w(s, u) + (K_u A_N)(s) \quad \text{for every feasible } u. \tag{F.6}$$

The first inequality rules out perpetual rejection. The second rules out all postponements, including saving to reduce future collection costs. It follows from $A_N - K_u A_N \geq L_1 \min\gamma - (1+d)K > 0$. The contraction of the augmented stopping operator makes A_N its value and immediate construction its unique timing choice. Subsequent governments face the same inequality if a low-capacity state is reached after a deviation. Country 1's access dominance holds for every current investment and finance choice, so the separate stopping verification also verifies its joint policy choice. Installed governments optimally continue the financing and access policies proved above.

For fixed endowments, one can instead impose (F.6) directly. It is precisely the strict fixed-income test, including real fiscal costs and the actual financial transitions; the endowment construction establishes their joint attainability.

Welfare and equilibrium completion. Both regimes have the same current physical allocation and quality, and the same strict active-research and open-access policies at a given capacity. The private regime remains at L. The public regime pays the engineering cost, builds, and incurs exactly the collection costs already included in C_G. Summing domestic budgets cancels the proceeds and payments of the claims as well as domestic taxes. For the same initial operating and financial state,

$$W_1^G(s) - W_1^P(s) = A_N(s) > 0.$$

The welfare gain counts operating income net of engineering and collection resources. Research, access, construction, tax collection, claim pricing and dated market clearing have now been verified

on the same primitive region. The one-stage deviation principle, with bounded quality-scaled values, completes the stationary Markov-perfect equilibrium verification. The construction establishes the stated stationary equilibria.

All strict comparisons have positive uniform margins over the finite operating states and the compact financial state set. Small perturbations preserve the result whenever the contract correspondences, prices and continuation values vary continuously and fiscal feasibility retains slack. The same construction admits small positive home learning and home feedback rights by the full-state continuity argument following Corollary 1. Feedback entitlements remain specified by Λ, and welfare is measured by domestic consumption.

F.2 Proof of Corollary 2

Proof. Select the strict construction in Appendix F.1 with $c_G = 0$, a continuous convex collection-cost function, and a tax cap strictly above every required tax stream. Use the retained-earnings and construction-contingent claims institution of the model on a compact wealth interval. Public waiting budgets and feasible financial correspondences depend on w, R_L, the assigned tax bound, and the realized quality multiplier; they do not change with research costs. Operating probabilities do change. All resource and resident-funding inequalities can be made strict by the finite labor endowments already used in the construction.

Apply Proposition 7 to three capacity plans: permanent low capacity, permanent high capacity, and construction today. Choose the action enlargement small enough to preserve the resource inequalities for every deviation. The permanent-capacity kernels are M_L^τ and M_H^τ. The construction-date kernel M_T^τ uses low capacity and anticipates high capacity next date. All three kernels and their laboratory distributions converge uniformly.

For public current construction put

$$h^\tau = M_T^\tau (I - M_H^\tau)^{-1}\mathbf{1}, \qquad n^{*,\tau}(x,w) = \frac{[J - w - R_L - h^\tau(x)R_H]_+}{1 + h^\tau(x)}.$$

These functions, the issue prices and $C_G^\tau = (1 + h^\tau)\phi(n^{*,\tau})$ converge uniformly on the compact financial state set. Fiscal feasibility retains its strict slack. After issue, $n^{*,\tau}$ is carried as an inherited obligation. The conditional tax-smoothing proof applies to the new installed kernel. The strict access margins in Appendix F.1 persist uniformly over all such obligations. Current issue prices and already contracted payments are held fixed in a hidden unilateral operating deviation; equilibrium prices are recalculated for the perturbed research policies.

For investment verification, let u_T^τ be the current national operating income at low capacity under anticipated construction, excluding engineering and fiscal resource costs. Define

$$\begin{aligned} U_T^\tau &= (I - M_T^\tau)^{-1} u_T^\tau, \\ U_H^\tau &= (I - M_H^\tau)^{-1} u_H^\tau, \\ A_N^\tau(s) &= \left[M_T^\tau (U_H^\tau - U_T^\tau)\right](x) - J - C_G^\tau(s). \end{aligned} \tag{F.7}$$

Both u_H^τ and U_H^τ are gross of inherited tax-collection losses. All such losses, including the current loss, are subtracted once through C_G^τ. The reference U_T^τ keeps the current low-capacity operating policies fixed at every date. This reference subtracts a common flow from the government's deviation problem. A unilateral investment deviation cannot revise their already submitted current research; future low-capacity governments follow the candidate construction rule.

The obstacle A_N^τ converges uniformly on the compact financial state set. The wealth maps and waiting feasible sets are unchanged, so the quality-weighted waiting operators K_u^τ converge in operator norm, uniformly over feasible u and inherited states. Let $\delta_0 > 0$ be a common lower bound for the unperturbed positivity and waiting gaps in (F.6). With the unchanged waiting resource cost $c_w(s,u)$ and row-mass bound $d = \beta a < 1$,

$$\begin{aligned} A_N^\tau(s) + c_w(s,u) - (K_u^\tau A_N^\tau)(s) \geq \delta_0 - (1+d)\|A_N^\tau - A_N^0\|_\infty \\ - \sup_u \|(K_u^\tau - K_u^0) A_N^0\|_\infty > 0 \end{aligned} \tag{F.8}$$

for sufficiently small τ. Also $A_N^\tau \geq \delta_0 - \|A_N^\tau - A_N^0\|_\infty > 0$. Thus the perturbed obstacle is the fixed point of the public stopping operator, with immediate construction uniquely optimal. The uniform inequality verifies the continuous financial state problem. Installed fiscal controls are reoptimized by the explicit formula above.

The private construction-return envelope is computed from the private permanent-low and postconstruction continuations. With interior research, a construction deviation leaves current permanent-low effort fixed and uses the high-capacity continuation. Compactness of Θ and uniform convergence of these values preserve $J > \overline{B}^\tau$. Thus every admitted corporate controller continues to reject construction, even if its financing restrictions are relaxed. The private rejection is uniform over financial choices. The strict access comparisons also persist on this path.

The two national welfare values, computed under their respective operating policies and net fiscal costs, converge uniformly to their unperturbed values. Their originally strict difference remains positive. Each welfare value uses its own low-capacity research policy and construction timing. The stationary growth and leadership comparisons persist by Proposition 7. Finally, choose the original research bound sufficiently small on a smooth effective-capacity interval with $\mathcal{B}' > 0$, before

choosing τ. Corollary 3 then supplies the strictly positive state-by-state research response. All discrete policy inequalities and all continuous laboratory deviations are verified in the same modified economy. The conclusion is local to this strictly verified branch. □

G Net institutional returns and dynamic implementation

G.1 Equivalent forms of the national investment test

For a finite operating state and waiting kernel M_L, put $m = M_L\mathbf{1}$. The conditions in Proposition 2 can be written

$$C_G \le A^0, \qquad (I - M_L)C_G \le (I - M_L)A^0. \tag{G.1}$$

For a constant construction-date implementation cost $c \ge 0$, its maximum admissible value is

$$c^* = \min_x \left\{ A^0(x), \frac{[(I - M_L)A^0](x)}{1 - m(x)} \right\}. \tag{G.2}$$

When $\Gamma, \gamma > 0$, the equivalent labor-income requirement is

$$L_1 \ge \max_x \left\{ \frac{[J + C_G(x) - \mathcal{J}_C(x)]_+}{\Gamma(x)}, \frac{[(I - M_L)(J\mathbf{1} + C_G - \mathcal{J}_C)]_+(x)}{\gamma(x)} \right\}. \tag{G.3}$$

The two terms value construction against permanent rejection and against postponement. For a reference income $Y_1(x) > 0$, the first test is

$$\frac{L_1}{Y_1(x)}\Gamma(x) \ge \frac{J - \mathcal{J}_C(x) + C_G(x)}{Y_1(x)}. \tag{G.4}$$

These expressions allow project costs, corporate returns, and fiscal losses to be compared with the discounted labor-income gain at a fixed income base.

G.2 Proof of Proposition 2

Proof. The matrix M_L is nonnegative with row sums at most $d < 1$. Its option operator $Tv = \max\{0, A_N, M_L v\}$ is a contraction, with perpetual rejection available at value zero. If immediate construction is optimal at every state, the value is $W_N = A_N \ge 0$, and the Bellman inequalities give $A_N \ge M_L A_N$. Conversely, these inequalities make A_N the fixed point, with immediate construction an optimal policy. Strict inequalities rule out waiting and perpetual rejection. Substituting $A_N = A^0 - C_G$ proves (G.1); using $\gamma = (I - M_L)\Gamma$ gives (33).

For a constant cost c, positivity requires $c \le A^0(x)$ and the waiting comparison requires $c[1 - m(x)] \le [(I - M_L)A^0](x)$. Each denominator $1 - m(x)$ is strictly positive. Taking the minimum of these upper bounds proves (G.2). When $J > \overline{B}$, all admitted corporate obstacles are negative; their option value is zero even with unrestricted construction finance. On the augmented state space the operator is $Tv = \max\{0, A_N, \sup_u[-c_w(\cdot, u) + K_u v]\}$. Its modulus is at most $\beta a < 1$, and the same fixed-point argument applies on the invariant state set. □

G.3 Proof of Proposition 5

Proof. For bounded v, w, $\|\mathcal{K}v - \mathcal{K}w\|_\infty \le d\|v - w\|_\infty$. The operator $v \mapsto \max\{b, \mathcal{K}v\}$ is therefore a monotone contraction. Its bounded fixed point B^* is nonnegative and is the least function satisfying $B \ge b$ and $B \ge \mathcal{K}B$. For example, start at b and iterate the operator; each iterate increases and is bounded above by every such majorant, and convergence is uniform.

For any nonnegative grant schedule that implements immediate construction at every state, private option value is $B = A_P + \mathfrak{g}$. Immediate construction must be at least as good as never constructing and as every waiting choice; thus $B \ge 0$ and $B \ge \mathcal{K}B$. Nonnegative grants require $B \ge A_P$, and current funding requires $B \ge A_P + J - \mathcal{F}$. Consequently $B \ge b$, so $B \ge B^*$ and $\mathfrak{g} \ge B^* - A_P$ pointwise.

Conversely, $\mathfrak{g}^* = B^* - A_P \ge 0$ satisfies funding, and the private option operator under this schedule has fixed point B^* because $B^* \ge \mathcal{K}B^*$. Immediate construction is optimal everywhere. For $\mathfrak{g}^* + \epsilon$, candidate value is $B^* + \epsilon$. Since all kernel row masses are at most d,

$$B^* + \epsilon - \mathcal{K}(B^* + \epsilon) \ge \epsilon(1 - d) > 0.$$

This value is positive and exceeds every waiting choice, proving strict private implementation.

For the government comparison, hold future $\mathfrak{g}^*$ fixed. Because $\mathcal{K}B^* \ge 0$, the fixed-point equation is equivalent to

$$\mathfrak{g}^*(s) = \max\{0, J - \mathcal{F}(s), (\mathcal{K}B^*)(s) - A_P(s)\}.$$

Thus $\mathfrak{g}^*$ is the minimum current offer that induces and funds current construction given the future rule. A smaller offer leaves either funding or the construction incentive unsatisfied; a larger offer weakly raises fiscal resource costs. The national obstacle inequalities rule out waiting in favor of the minimum implementing offer. Recipient acceptance at indifference gives the stated sequential government strategy. Future fiscal servicing and operating choices remain subject to their own optimality conditions. The ϵ modification gives strict private incentives at a higher fiscal cost than the minimum discretionary offer. □

G.4 Household contributions and the contract boundary

The contribution game allows workers to supply current resources. The sponsor holds a given corporate portfolio. Worker household j receives labor income from its endowment L_j and holds no other claims whose payoffs are affected by construction. Corporate claims are held by a separate group of residents. Corporate owners and worker households together exhaust domestic ownership, with $\sum_j L_j = L_1$. Worker households can make unilateral payments conditional on current construction; offers become payable only upon current construction and expire otherwise. The offers and construction decisions recur each date. Current research and access choices are submitted before contributions and are not observed by contributors. Only unilateral contribution contracts are enforceable in this game.

Lemma 1 (Dispersed current contributions)**.** *Suppose the sponsor's construction obstacle is strictly negative at every state and write*

$$\delta(x) = J - \mathscr{F}[y^P](x) > 0.$$

If

$$L_j \Gamma(x) < \delta(x) \quad \textit{for every household } j \textit{ and state } x, \tag{G.5}$$

then zero contributions and no construction form a stationary equilibrium of the contribution and investment subgame, even when $L_1\Gamma(x) > \delta(x)$ at some states. A financing restriction weakly raises the transfer needed to induce construction, preserving the zero-contribution equilibrium.

Proof. Specify zero contributions after every future unbuilt history. Without a current transfer, the sponsor strictly rejects construction in every state. An offer that induces current construction must cover at least its return deficit $\delta(x)$; meeting a dated funding constraint can require more. With all other offers zero, the contributing worker captures at most $L_j\Gamma(x)$ and pays the whole inducing transfer. Condition (G.5) makes the deviation unprofitable. After a smaller offer, the sponsor continues to reject construction and the offer expires. The sponsor chooses an optimal response after every off-path vector of offers. These strategies verify the subgame at every state. Aggregate worker gains can exceed the construction deficit while every individual gain remains below it. □

The zero-contribution equilibrium follows the noncooperative public-good logic of Bergstrom et al. (1986). An enforceable collective agreement must use the actual participation values, including future renegotiation, and can enlarge the implementation set. If one household controls the sponsor and internalizes all the excluded income, the relevant construction gain is $L_1\Gamma - \delta$. Household contracts and control rights therefore determine the implementation set.

H Pledgeability and public finance

H.1 Proof of Proposition 4

Proof. If the organization sells claims worth f, its current distribution is $w + S_L - J + f$ and its retained continuation value is $H - f$. Their sum is $w + S_L - J + H$. A nonnegative current distribution can be obtained for some $f \in [0, \kappa^p H]$ exactly when (41) holds. Subtracting the continuation value of permanent low capacity gives (42). Expanding these expressions proves the first statement.

For $h > l$, the slopes of the two boundaries are respectively

$$-\frac{\Delta\Pi\, h}{\Pi_L(h-l)}, \qquad -\frac{\kappa^p \Delta\Pi\, h}{\Pi_L(1+\kappa^p h)}. \tag{H.1}$$

The first is strictly more negative because $1 + \kappa^p h > \kappa^p (h - l)$, or $1 + \kappa^p l > 0$. At their intersection, $hS_H = J + lS_L$; substituting into the funding equality gives (44).

With $\kappa^p = 1$,

$$\mathcal{F} - \mathcal{J} = w + (1 + l)S_L \geq 0, \tag{H.2}$$

with a strict inequality when $S_L > 0$. For a general waiting option, let $W \geq 0$ denote its value after subtracting the common current cash $w + S_L$. Current construction can be optimal only if $H - J \geq W$. Then $w + S_L + H \geq J + w + S_L + W \geq J$, so selling sufficiently many future claims funds the choice. The waiting value in this argument includes every feasible retained-earnings choice and the associated continuation policy. □

H.2 Current construction grants

Proposition 10 (Implementation by a contingent grant)**.** *The smallest grant that makes current construction both privately acceptable and financeable is the amount in* (45)*.*

If construction-contingent transfers are enforceable and the government can supply this amount, public finance with private control implements the same current physical allocation as direct public investment. At a binding incentive constraint, implementation is weak; a strictly larger feasible grant makes current construction strictly preferable. An anticipated change in future grants changes W and must be included in this comparison. Fiscal costs and continuation policies enter the government implementation conditions in Proposition 5.

H.3 Proof of Proposition 10

Proof. A current construction-contingent transfer raises the construction value from $w + S_L - J + H$ to $w + S_L - J + H + \mathfrak{g}$, while the no-construction value remains $w + S_L + W$. The incentive constraint is therefore $J - \mathfrak{g} \leq \Omega$. Available construction resources also rise by $\mathfrak{g}$, giving $J - \mathfrak{g} \leq \mathcal{F}$. Their joint minimum is (45). For strict implementation one can add an arbitrarily small positive amount when an incentive constraint binds; at the boundary, construction is an optimal choice. □

H.4 Growth, fiscal backing and collection costs

Public backing can use a source of repayment that is unavailable to a corporation: enforceable claims on an assigned tax base. We consider the public counterpart of the revenue-claim financing in Proposition 4. Claims may be contingent on the realized quality level. They are legally enforceable, trade at the household's valuation, and commit only the promised payments; subsequent governments continue to choose tax collection and operating policy subject to inherited obligations.

Fix a construction state $s = (L, x, b)$ and the current and installed operating kernels. Write $J_G(s) = J + c_G(s)$ for the total engineering and public implementation outlay, and suppress its state argument below. Let $h(x) = (M_0 G_H)(x)$ as in (39), and put

$$\mathcal{G}(x) = 1 + h(x). \tag{H.3}$$

This is the current value of one unit of receipts per unit of quality collected from the present date onward, under current construction. Suppose the government can pledge the unencumbered infrastructure receipts $R_H Q$ after installation and can supply liquid resources w together with current infrastructure receipts R_L. Its residual fiscal requirement is

$$\mathfrak{f}(s) = \left[J_G - w - R_L - h(x) R_H\right]_+. \tag{H.4}$$

When $\mathfrak{f} = 0$, full project-income capitalization covers the resource requirement. A public guarantee that bridges only a private pledgeability shortfall can then be serviced entirely from project income.

Let $n_t \geq 0$ denote net tax receipts per unit of contemporaneous quality, and let $\phi(n_t)$ be their real collection cost per unit of quality. Assume $\phi(0) = 0$ and ϕ is increasing and convex. The net tax base permits $n_t \leq \overline{n}$. The government can issue claims against the assigned receipts, subject to its intertemporal budget and the no-Ponzi condition. Convex collection costs create a tax-smoothing motive (Barro, 1979); here the endogenous quality process determines the income weights in that problem.

Proposition 11 (The capitalization of fiscal capacity)**.** *At the fixed operating kernels, construction can be supported by the tax base if and only if*

$$\mathfrak{f}(s) \le \mathcal{G}(x)\overline{n}. \tag{H.5}$$

Among admissible tax policies funding this residual requirement, the minimum expected present value of real collection costs is

$$\mathcal{D}(\mathfrak{f};\mathcal{G}) = \mathcal{G}\,\phi\left(\frac{\mathfrak{f}}{\mathcal{G}}\right). \tag{H.6}$$

It is attained by constant normalized net receipts $n_t = n^* = \mathfrak{f}/\mathcal{G}$, including at the construction date. The total implementation resource cost beyond engineering expenditure is $c_G(s) + \mathcal{D}(\mathfrak{f};\mathcal{G})$. The payment obligation n^*Q_t can be assigned to investors; subsequent governments optimally maintain this collection path, conditional on the operating kernels and the inherited claims.

If ϕ is differentiable, increasing the capitalization factor at a fixed fiscal requirement weakly lowers its collection cost:

$$\frac{\partial \mathcal{D}}{\partial \mathcal{G}} = \phi(n^*) - n^*\phi'(n^*) \le 0. \tag{H.7}$$

Where $\mathfrak{f} = J_G - w - R_L - hR_H > 0$, an increase in h also capitalizes infrastructure receipts, giving

$$\frac{d\mathcal{D}}{dh} = \phi(n^*) - (n^* + R_H)\phi'(n^*) \le 0. \tag{H.8}$$

These derivatives hold the total construction outlay J_G and the specified receipt levels fixed. When the project also changes those objects, their funding effects enter the total collection-cost derivative derived in Appendix H.6.

Growth affects the financing comparison through both private income claims and the assigned fiscal base. A higher expected quality path enlarges the value of tax receipts and can spread a given net requirement across a larger future income base. Public and private claims use the same household discount factor. Collection losses apply to the actual residual fiscal requirement in (H.4), while a guarantee that only expands pledgeability can be serviced from project income.

The financial contract in this result is explicit. Fully enforceable quality-contingent tax claims allow the government to use the valuation $\mathcal{G}$. A restriction to a fixed state-independent repayment would additionally require its pathwise repayment constraints. In an equilibrium application, the installed operating policy must remain optimal with the inherited tax obligation; the national flow then includes $-\phi(n^*)$.

Without innovation, $\mathcal{G} = 1/(1-\beta)$ and $h = \beta/(1-\beta)$. The result reduces to

$$\mathfrak{f} = \left[J_G - w - R_L - \frac{\beta R_H}{1-\beta}\right]_+, \qquad \mathcal{D}(\mathfrak{f}) = \frac{\phi((1-\beta)\mathfrak{f})}{1-\beta}. \tag{H.9}$$

This deterministic case separates a credit restriction from an intertemporal resource deficit. A face larger than one period's project receipts can still be serviced without taxes when future project income supports its interest and refinancing.

H.5 Proof of Proposition 11

Proof. For a history-dependent normalized payment z_t, define its current valuation by

$$\mathbb{V}_x[z] = \mathbb{E}_x \sum_{t\geq 0} \beta^t \frac{Q_t}{Q_0} z_t, \qquad \mathbb{V}_x[1] = \mathcal{G}(x).$$

The expectation uses the specified current and installed operating policies. The quality-weighted kernels have row sums bounded strictly below one, so these valuations are finite for bounded payments. The receipts that can be supplied without taxes have value $w + R_L + hR_H$. Summing dated public budgets and using the no-Ponzi condition therefore gives $\mathbb{V}_x[n] \geq \mathfrak{f}$. Since collection costs are increasing, equality is optimal whenever $\mathfrak{f} > 0$.

The measure placing weight $\beta^t(Q_t/Q_0)/\mathcal{G}(x)$ on each dated history is a probability measure after taking expectations. Jensen's inequality gives

$$\mathbb{V}_x[\phi(n)] \geq \mathcal{G}(x)\, \phi\left(\frac{\mathbb{V}_x[n]}{\mathcal{G}(x)}\right) \geq \mathcal{G}(x)\phi\left(\frac{\mathfrak{f}}{\mathcal{G}(x)}\right). \tag{H.10}$$

Constant normalized receipts $n^* = \mathfrak{f}/\mathcal{G}(x)$ attain the bound. They satisfy the tax-capacity restriction exactly when $\mathfrak{f} \leq \mathcal{G}(x)\overline{n}$. The latter condition is also necessary, since the value of any bounded admissible tax stream is at most $\mathcal{G}(x)\overline{n}$.

For dated implementation when $\mathfrak{f} > 0$, collect n^* currently and sell enforceable claims on subsequent receipts $(R_H + n^*)Q_t$. Their competitive value is $h(R_H + n^*)$. Current funding is exact:

$$w + R_L + n^* + h(R_H + n^*) = w + R_L + hR_H + (1 + h)n^* = J_G.$$

After installation, project receipts meet the $R_H Q_t$ portion of the assigned claims and tax receipts meet the $n^* Q_t$ portion. These assigned income streams cover the contracted repayments. The government retains operating authority subject to those obligations. When $\mathfrak{f} = 0$, a sufficiently small share of project receipts finances the required shortfall with zero tax collection.

The constant collection path is sequentially optimal conditional on those claims and the operating kernels. At an installed history, let $G_H(x_t)$ be the remaining capitalization factor. The inherited tax-payment obligation has value $n^* G_H(x_t)$. Project income is assigned to the corresponding project-income claim. Any tax-and-refinancing plan honoring the residual obligation has normal-

ized tax value at least $n^* G_H(x_t)$. Applying the same Jensen inequality at that history gives a collection cost at least $G_H(x_t)\phi(n^*)$, attained by continuing n^*. Continuation at n^* attains this lower bound across feasible refinancing plans. This argument verifies tax choices given inherited enforceable claims; operating-policy optimality is evaluated with the resulting flow cost $\phi(n^*)$.

Differentiating $\mathcal{G}\phi(\mathfrak{f}/\mathcal{G})$ at fixed $\mathfrak{f}$ proves (H.7), since convexity and $\phi(0) = 0$ imply $\phi(n) \leq n\phi'(n)$. Where the fiscal gap is positive, $\mathfrak{f}'(h) = -R_H$ and $\mathcal{G}'(h) = 1$, so

$$\frac{dn^*}{dh} = -\frac{R_H + n^*}{1 + h}.$$

The chain rule gives (H.8). If the gap reaches zero, zero collection remains feasible.

Finally, no innovation gives $h = \beta/(1-\beta)$ and the stated deterministic formula. Its dated budgets also admit a conventional constant-face implementation. Write $A = J_G - w - R_L$. For $\mathfrak{f} > 0$, collect $n = (1-\beta)\mathfrak{f}$ and issue face $F = A + R_H$. Then $n + \beta F = A$, and each subsequent budget satisfies $R_H + n - (1-\beta)F = 0$. The discounted terminal face vanishes. For $\mathfrak{f} = 0$, issue $F = [A]_+/\beta \leq R_H/(1-\beta)$; rolling this face requires no taxes because $R_H - (1-\beta)F \geq 0$. □

H.6 Project changes and fiscal collection costs

The derivatives in (H.7)–(H.8) hold the total construction outlay and the specified receipt levels fixed. If a project parameter ξ also changes those objects, write $n = (J_G - w - R_L - hR_H)/(1+h) > 0$, with $J'_G = J' + c'_G$. The total collection-cost response is

$$\frac{d\mathcal{D}}{d\xi} = \phi'(n)[J'_G - w' - R'_L - hR'_H] + h'\{\phi(n) - (n + R_H)\phi'(n)\}. \tag{H.11}$$

Thus higher capitalization weakly lowers collection costs at a fixed funding requirement, but a larger engineering or implementation outlay can offset that effect when the project itself changes. At the boundary where the untruncated fiscal gap is zero, the one-sided derivative in a direction with gap change $\mathfrak{f}'$ is $\phi'_+(0)\max\{\mathfrak{f}', 0\}$; inside the zero-gap region no taxes need be collected. The formula follows by differentiating $(1+h)\phi(\mathfrak{f}/(1+h))$ and applying the positive part to the fiscal gap. The corresponding change in total implementation resource costs is $c'_G + d\mathcal{D}/d\xi$.

I Investment timing and finite-horizon outcomes

The return and funding frontiers enter a dynamic investment problem. Optimal current construction requires a favorable comparison with both perpetual rejection and waiting, because future feedback

and ownership can change the project's value. This section states the timing problem and distinguishes permanent nonadoption from a finite delivery delay.

I.1 A financing-constrained stopping problem

Fix operating rules at each capacity that depend only on the operating state x. The low-capacity rule is the action prescribed before construction, including any anticipation of delivery. Continue that rule to define $G_L = (I - M_L)^{-1}\mathbf{1}$, and let G_H use the installed rule. For the nonnegative infrastructure and ordinary-profit receipts S_k in (40), define

$$A(x) = \left[M_L(S_H G_H - S_L G_L)\right](x) - J. \tag{I.1}$$

Current construction changes next-period capacity, leaving the already submitted current operating choices unchanged. This is the appropriate deviation comparison even when the low-capacity rule reflects anticipated delivery.

Under construction-contingent revenue issuance, let $W(x, w)$ be the controller's option value after subtracting its liquid assets and the value of perpetual low-capacity receipts. The recursive investment problem is

$$W(x,w) = \max\left\{ \begin{array}{ll} A(x) & \text{if } J \le w + S_L + \kappa^p S_H(M_L G_H)(x), \\ \sup\limits_{d \in \mathcal{D}(w)} \beta \mathbb{E}\left[a^\zeta W\left(x', \dfrac{w + S_L - d}{\beta a^\zeta}\right) \middle| x \right] & \end{array} \right\}. \tag{I.2}$$

Here d is a current dividend. The financing institution imposes $w \in [0, \bar{w}]$ with $\bar{w} \ge J$. Feasible dividends obey $\max\{0, w + S_L - \beta\bar{w}\} \le d \le w + S_L$, so next-period normalized wealth remains in this interval under either innovation outcome. This restriction bounds domestic financial calls; construction and saving are evaluated within the stated institution. The expectation uses the physical transition and innovation outcome generated by the low rule.

Proposition 12 (The investment option with retained earnings)**.** *Equation* (I.2) *has a unique bounded nonnegative solution. It gives the controller's complete best response to the prescribed operating rules. If* $A < 0$ *at every operating state, it never constructs, regardless of pledgeability. If construction is financeable at every state under consideration and* $A > 0$ *and* $A > M_L A$*, immediate construction is optimal there, provided that set is invariant under every feasible waiting choice. Otherwise the equation determines the value of waiting jointly for liquid resources and a favorable operating state.*

Proof of Proposition 12. Under construction, competitively priced revenue issuance changes the current payment and retained future value by equal and opposite amounts. Subtracting $w+S_L G_L(x)$ from the resulting value gives $A(x)$. Under waiting, next liquid wealth is $w' = (w+S_L-d)/(\beta a^{\zeta})$, so $\beta\mathbb{E}[a^{\zeta}w'] = w+S_L-d$. The current dividend and the continuation value of wealth therefore cancel from the option comparison. The remaining low-income terms cancel because $G_L = \mathbf{1} + M_L G_L$. This proves (I.2).

Its operator maps bounded functions into bounded functions and is a contraction in the sup norm with modulus at most $\beta a < 1$. Choosing never to construct gives zero option value. The fixed point is therefore nonnegative and bounded above by $\max_x [A(x)]_+$. If $A < 0$ everywhere, zero is the unique fixed point and construction is strictly suboptimal. If the stated immediate-construction inequalities hold, $W(x,w) = A(x)$ satisfies the equation on the invariant set, because every waiting choice then has value $(M_L A)(x) < A(x)$. Contraction and the one-stage deviation principle establish optimality. Attainment follows from upper semicontinuity, which allows the value to jump upward at a financing threshold. Start value iteration at zero. Construction feasibility is closed in wealth, the feasible dividend correspondence is compact and continuous, and the next-wealth map is continuous. The operator therefore preserves upper semicontinuity. Its uniform limit is upper semicontinuous, and the waiting maximum is attained on the compact dividend set. □

Long-run shares and finite policy horizons. The speed of ownership adjustment from a given initial condition depends on displacement probabilities. A useful bound uses the maximum probability of displacing the initial owner.

Proposition 13 (Ownership over a finite horizon)**.** *Compare two institutional or capacity paths from the same initial owner i. Suppose, in either path and at every history before the first ownership change, the conditional probability that the other laboratory innovates is at most $\bar{s}_i < 1$. For any integer $T \geq 0$,*

$$\left| \overset{G}{\Pr}(\ell_T = 1) - \overset{P}{\Pr}(\ell_T = 1) \right| \leq 1 - (1-\bar{s}_i)^T \leq T\bar{s}_i. \tag{I.3}$$

On the active-research branch, one may take, for $j \neq i$,

$$\bar{s}_i = \frac{\eta\chi_j(1+\nu_j)}{1+\eta\chi_i+\eta\chi_j(1+\nu_j)}. \tag{I.4}$$

The bound permits a construction delay, stochastic feedback and different transition laws under the two operating processes.

For a fixed number of model dates, the ownership-probability difference therefore vanishes as η tends to zero. For the comparison of fixed installed capacities, this can occur even when the

limiting stationary share difference does not vanish. A nonvanishing ownership effect requires a horizon at least of order $1/\eta$ in this family. Taking the long-run limit before making innovation rare is consequently different from reversing those limits. Appendix I.4 gives exact finite-horizon formulas for ownership, invention counts, expected quality, and consumption welfare, with the construction-date kernel distinguished from the installed kernel. Model dates have no calendar-year interpretation without an empirical time normalization.

I.2 Arrival rates and research responses

When the nonrecipient has constant effective intensity z, the contest implies the pointwise identity

$$q_{\text{nonrecipient}}(x) = \frac{z}{1 + h_{\text{recipient}}(x) + z} = z[1 - q(x)].$$

Taking stationary expectations proves (D.6), using $\bar{q} = D/Z$ and $Z - D = C + f$. Differentiating, with x, y, z, δ, ω fixed, gives

$$\frac{d\bar{q}}{df} = \frac{C(y-x)}{Z^2}, \qquad \frac{d\bar{q}_{\text{recipient}}}{df} = (1+z)\frac{d\bar{q}}{df}, \qquad \frac{d\bar{q}_{\text{nonrecipient}}}{df} = -z\frac{d\bar{q}}{df}. \tag{I.5}$$

The same identity holds for expected counts over a finite horizon: $\mathbb{E}N_{\text{nonrecipient}}(T) = z[T - \mathbb{E}N(T)]$ whenever z is constant along the path.

Fix a positive η and a capacity point with a strictly positive feedback response. In the interior extension, the invariant distribution and the state-specific innovation probabilities converge in C^1. Their product $\bar{q}_i$ therefore converges in C^1 as well. The strictly negative derivative in (I.5) persists for every sufficiently small positive cost perturbation τ. This conclusion is compatible with the positive effort response in Corollary 3. With larger cost perturbations, z can vary with state and capacity, so the innovation-rate response also depends on endogenous changes in that intensity. For any differentiable equilibrium branch with positive innovation rates,

$$d\log \bar{q}_i = d\log \rho_i + d\log \bar{q}.$$

Domestic invention counts can then rise while their share falls if world invention counts grow sufficiently faster.

I.3 Proof of Proposition 13

Let σ be the first date on which a rival innovation displaces the initial owner, and set $\sigma = \infty$ if no such event occurs. Conditional on survival up to any date, the next survival probability is at least $1 - \bar{s}_i$. Iterated conditioning gives

$$\overset{k}{\Pr}(\sigma > T) \geq (1 - \bar{s}_i)^T, \qquad k \in \{G, P\}.$$

Being outside the initial owner's country at date T requires a first displacement by that date. Thus the probability of being outside the initial owner lies in $[0, 1 - (1 - \bar{s}_i)^T]$ under each path. Their difference is bounded by the length of that interval, proving (I.3) with coefficient one. Before the first displacement, $h_i \geq \eta\chi_i$ and $h_j \leq \eta\chi_j(1 + \nu_j)$ on the active-research branch. Monotonicity of $h_j/(1 + h_i + h_j)$ establishes (I.4).

For interior research, the same proof applies if the two paths have common positive effort bounds $\underline{r}_i \leq r_i(x)$ and $r_j(x) \leq \overline{r}_j$. Replace (I.4) by

$$\bar{s}_i = \frac{\chi_j(1 + \nu_j)\overline{r}_j}{1 + \chi_i\underline{r}_i + \chi_j(1 + \nu_j)\overline{r}_j}.$$

For a common random initial owner, average the corresponding bounds using its initial probabilities. The resulting assertion applies to paths sharing the same initial ownership distribution.

If an ownership-probability difference is at least $b \in (0, 1)$, the bound requires

$$T \geq \frac{\log(1 - b)}{\log(1 - \bar{s}_i)}.$$

This gives a necessary lower bound on the horizon for such an ownership effect; its attainment depends on the transition laws. When $\bar{s}_i = O(\eta)$ with a positive first-order coefficient, it is of order $1/\eta$. Compare fixed installed capacities H and L. If their stationary frontier ownership difference has a nonzero limit $\Delta\theta_0$ in the family of Appendix J, irreducibility at every positive η and the finite-horizon bound give the distinct iterated limits

$$\lim_{\eta \downarrow 0} \lim_{T \to \infty} [\overset{H}{\Pr}(\ell_T = 1) - \overset{L}{\Pr}(\ell_T = 1)] = \Delta\theta_0, \qquad \lim_{T \to \infty} \lim_{\eta \downarrow 0} [\overset{H}{\Pr}(\ell_T = 1) - \overset{L}{\Pr}(\ell_T = 1)] = 0.$$

These limits hold the capacity paths fixed. Applying them to endogenous investment as η vanishes also requires the institutional construction conditions to hold at the specified labor endowments.

I.4 Exact outcomes along the construction path

Let P_L, P_H be physical operating matrices under permanent low and high capacity, and let P_T be the current physical matrix when low capacity is followed by installation next date. Let q_i^L, q_i^H, q_i^T be their country-specific innovation-probability vectors. These are the three separately solved operating environments in the interior model. For $T \geq 1$, a common initial operating distribution μ_0, and $e_i(x) = \mathbf{1}_{\{\ell=i\}}$, the ownership probabilities are

$$p_i^P(T) = \mu_0 P_L^T e_i, \qquad p_i^G(T) = \mu_0 P_T P_H^{T-1} e_i. \tag{I.6}$$

Expected invention counts through date $T-1$ are

$$\begin{aligned} \mathbb{E}N_i^P(T) &= \mu_0 \sum_{t=0}^{T-1} P_L^t q_i^L, \\ \mathbb{E}N_i^G(T) &= \mu_0 q_i^T + \mu_0 P_T \sum_{t=0}^{T-2} P_H^t q_i^H, \end{aligned} \tag{I.7}$$

where the second sum is empty when $T = 1$.

Expected quality uses the quality-weighted transition matrix. Put $F_k = M_k/\beta$, $k \in \{L, H, T\}$. Then

$$\mathbb{E}^P \frac{Q_T}{Q_0} = \mu_0 F_L^T \mathbf{1}, \qquad \mathbb{E}^G \frac{Q_T}{Q_0} = \mu_0 F_T F_H^{T-1} \mathbf{1}. \tag{I.8}$$

The distinction accounts for the correlation of quality growth and operating states.

For the national welfare comparison, fix an initial state (x, w), including the initial ownership and wealth. Let $n^*(x, w)$ be the normalized tax obligation issued at construction. For the fixed operating policies in this comparison, let u_T^{op} and u_H^{op} be national operating income under the construction-period and post-construction policies, respectively, net of research resource costs. The national consumption flows are

$$\begin{aligned} u_T^G(x, w) &= u_T^{\mathrm{op}}(x) - J - c_G(x, w) - \phi(n^*(x, w)), \\ u_H^G(y; n^*) &= u_H^{\mathrm{op}}(y) - \phi(n^*). \end{aligned}$$

Thus construction uses engineering and public implementation resources, and the inherited tax obligation entails a real collection cost in each subsequent period. Let u_L^P be national income under

the nonconstructing private equilibrium. Domestic debt service and other transfers cancel in these flows. With δ_x the row point mass at x,

$$\begin{aligned} W_1^P(T) &= \delta_x \sum_{t=0}^{T-1} M_L^t u_L^P, \\ W_1^G(T) &= u_T^G(x,w) + \delta_x M_T \sum_{t=0}^{T-2} M_H^t u_H^G(\cdot; n^*). \end{aligned} \tag{I.9}$$

These values are per unit of initial quality and use the separate construction-date, installed-capacity and permanent-low research policies. If national flows are bounded in absolute value by $\overline{u}_P, \overline{u}_G$, respectively,

$$\left|[W_1^G - W_1^P] - [W_1^G(T) - W_1^P(T)]\right| \le \frac{(\overline{u}_G + \overline{u}_P)(\beta a)^T}{1 - \beta a}. \tag{I.10}$$

This controls truncation of the discounted comparison at the same model horizon used for the other outcomes.

The first-displacement bound can also be evaluated exactly. Let $S_{i,k}$ be the restriction of P_k to transitions within the initial owner's states, including that owner's further innovations. For an initial feedback distribution μ_i, the probabilities of no first displacement by date T are

$$\mu_i S_{i,L}^T \mathbf{1}, \qquad \mu_i S_{i,T} S_{i,H}^{T-1} \mathbf{1}.$$

The absolute difference in date-T ownership is bounded by the larger of the two complementary probabilities. This is a sharper, transition-specific version of (I.3).

I.5 A two-state formula for finite invention counts

On the one-recipient active-research branch at a fixed capacity, set

$$u = \frac{f}{1 + x + z}, \qquad v = \frac{C}{1 + y + z}, \qquad p_* = \frac{u}{u + v}, \qquad \lambda_d = 1 - u - v.$$

If p_0 is the initial probability of validated feedback, then $p_t = p_* + (p_0 - p_*)\lambda_d^t$. For either laboratory let $q_i(0), q_i(1)$ denote its innovation probabilities in the two feedback states. Direct summation yields the exact transient formula

$$\mathbb{E}N_i(T) = T\bar{q}_i + (p_0 - p_*)[q_i(1) - q_i(0)]\frac{1 - \lambda_d^T}{1 - \lambda_d}. \tag{I.11}$$

Here $|\lambda_d| < 1$, although λ_d need not be positive. The expected feedback transient and the slow change of frontier ownership are distinct: feedback can stabilize while rare innovations still leave ownership close to its initial country.

J Small research intensity and discounted investment gains

J.1 Innovation intensity and the value of earlier adoption

The intensity of research determines the timing and magnitude of the quality gain. Using the home-learning labels of Proposition 1, write $x = \eta\alpha$, $y = \eta b$, and $z = \eta c$, with $0 < \alpha < b$ and $c > 0$. Hold β, a, δ, physical capacities, and learning primitives fixed as η varies, and let f_0 be the zero-training validation probability. Then

$$\rho_1, \theta_1 \longrightarrow \frac{\alpha\delta + b f_0}{(\alpha + c)\delta + (b + c) f_0}, \tag{J.1}$$

$$\rho_1 - \theta_1 = \eta \frac{\omega f_0 c \delta (b - \alpha)^2}{(\delta + f_0)\{(\alpha + c)\delta + (b + c) f_0\}^2} + O(\eta^2). \tag{J.2}$$

The foreign-feedback case follows by interchanging country labels. An expansion can therefore have a nonvanishing effect on either leadership share, including a majority reversal, even as research becomes rare. In contrast, the difference between the two shares shrinks, as does the interval separating their majority thresholds.

The distinction also matters for investment. The long-run growth rate and the discounted quality gains satisfy $g = O(\eta)$, $\Gamma = O(\eta)$, and $\gamma = O(\eta)$ when the other primitives are fixed. The expansions below give their strictly positive leading coefficients for an effective feedback expansion. If the part of the project cost left uncovered by corporate returns remains bounded away from zero, the labor-income base needed to cover it is of order at least $1/\eta$. A larger labor-income base compensates for the smaller discounted quality gain along this family. The investment thresholds in Proposition 2 make the required benefit explicit at each finite research intensity.

Use the one-recipient active-research branch, allowing the recipient to be in either country. With the notation of (J.1), put $t = (\alpha + c, b + c)'$ and let f_k^0 be validation at capacity k when training uses no inputs. Define

$$T_k = \begin{pmatrix} 1 - f_k^0 & f_k^0 \\ \delta & 1 - \delta \end{pmatrix}, \qquad H_k = \frac{\beta(a - 1)}{1 - \beta}(I - \beta T_k)^{-1} t. \tag{J.3}$$

On the two feedback states, the quality value has the expansion

$$G_k = \frac{\mathbf{1}}{1-\beta} + \eta H_k + O(\eta^2). \tag{J.4}$$

To prove it, the zero-research weighted kernel is βT_k. Its resolvent is nonsingular since $\beta < 1$. The derivative of its row-sum vector at zero is $\beta(a-1)t$, because the innovation probabilities are $\eta t_d + O(\eta^2)$. Differentiating $(I - M_k)G_k = \mathbf{1}$ gives (J.3). Changes in training-induced validation and innovation-contingent retention multiply a constant zero-research continuation and hence do not enter this first-order coefficient. The argument applies for every fixed $\omega \in [0,1]$.

Write $K = \beta(a-1)/(1-\beta)$, $R(f) = 1 - \beta + \beta(f+\delta)$, and $\Delta t = b - \alpha > 0$. The coefficient's components obey

$$H_1 - H_0 = \frac{K\Delta t}{R(f)}, \qquad H_0 = \frac{Kt_0}{1-\beta} + \frac{\beta f K \Delta t}{(1-\beta)R(f)}.$$

Their derivatives are

$$\frac{dH_0}{df} = \frac{\beta K \Delta t(1-\beta+\beta\delta)}{(1-\beta)R(f)^2} > 0, \qquad \frac{dH_1}{df} = \frac{\beta^2 \delta K \Delta t}{(1-\beta)R(f)^2} > 0. \tag{J.5}$$

For $0 < f_L^0 < f_H^0 < 1$, the construction and acceleration gains satisfy

$$\Gamma = \eta\, \beta T_L (H_H - H_L) + O(\eta^2), \tag{J.6}$$

$$\gamma = \eta\, \beta^2 T_L (T_H - T_L) H_H + O(\eta^2). \tag{J.7}$$

The first expression follows from $\Gamma = M_L(G_H - G_L)$. For the second use $\gamma = M_L(M_H - M_L)G_H$. The two regimes have the same innovation intensities at each fixed feedback state. Consequently $(M_H - M_L)\mathbf{1} = 0$ at every η, and the first nonzero term is the one displayed. Both coefficient vectors are strictly positive: (J.5) handles the first, while $(T_H - T_L)H_H$ is positive in the unvalidated state and zero in the validated state, which has positive probability δ of becoming unvalidated.

The stationary formulas in (C.2) give (J.1)–(J.2) by canceling the powers of η and expanding their nonzero remaining denominators. They also give

$$g = \eta \log a\, \frac{(\alpha+c)\delta + (b+c)f_0}{\delta + f_0} + O(\eta^2). \tag{J.8}$$

If f varies with training, $f = f_0 + O(\eta)$, which leaves the first-order share-gap and growth coefficients unchanged. For fixed baseline research productivities satisfying $\alpha < c < b$, the common limiting majority boundary is $f_*^0 = \delta(c-\alpha)/(b-c)$. Its share derivative is strictly positive. Equation (J.2) and the implicit function theorem therefore imply $f_\theta^* - f_\rho^* = O(\eta)$ when $\omega > 0$.

Finally, suppose at some initial state the uncovered national resource cost $J + C_G - \mathcal{J}_C$ has a strictly positive lower bound along the family. Construction requires $L_1\Gamma \geq J + C_G - \mathcal{J}_C$. Equation (J.6) then implies $L_1 = \Omega(1/\eta)$. This is a necessary scaling statement for that specified family, rather than a restriction on every finite-intensity equilibrium. In particular, a changing project cost or corporate return changes this comparison.

K Numerical methods and verification

K.1 Primitives and the comparison

Section 5 uses the quadratic research labor requirement specified there, with feasible effort $0 \leq r \leq .6$. The following primitives are held fixed across feedback assignments:

$$\begin{gathered}\beta = .9, \quad a = 1.05, \quad (\chi_1, \chi_2) = (1, 1.2), \quad \nu_1 = \nu_2 = 1, \\ \delta_1 = \delta_2 = .25, \quad \omega = .5, \quad \varepsilon_T = .001, \quad D_1 = D_2 = 1.05, \quad \vartheta_1 = \vartheta_2 = .2, \\ (L_1, L_2) = (1000, 100), \quad J = 30, \quad \phi(n) = .01n^2, \quad 0 \leq n \leq 1, \quad 0 \leq w \leq J.\end{gathered} \tag{K.1}$$

Both countries have ordinary-sector coefficient $b_i = 1$. Country 1's electricity and compute capacities are $(E_L, K_L) = (2, 1)$ and $(E_H, K_H) = (3.44, 2)$; country 2 has $(E_2, K_2) = (2, 1)$. The competitive allocation gives

$$\bar{e}_1^L = 1, \quad \bar{e}_1^H = 2, \quad \bar{e}_2 = 1, \quad (R_L, R_H) = (1.5, 2.6), \quad (\Pi_L, \Pi_H) = (.5, .6), \quad R_2 + \Pi_2 = 2.$$

These receipts follow from competitive market clearing. For example, ordinary electricity use is 1 at low capacity and 1.44 at high capacity; the corresponding electricity prices are .5 and $5/12$, while the sum of electricity and compute prices is one in both cases.

The feedback matrix varies over the 99 assignments in (36). Both labor endowments and the engineering cost remain fixed. The engineering cost is three percent of the domestic labor-income flow, and background labor is the largest component of national income at the assigned primitives.

K.2 Research and continuous-action verification

For each σ, the state is (ℓ, d_1, d_2), with eight possible values. At each installed capacity, the laboratory value system and its sixteen first-order conditions are solved jointly. At the construction date, current inputs are those of low capacity and continuation values are the installed high-capacity values. This gives three separate action profiles r_L, r_H, r_T. A deviating private controller takes the submitted low-capacity profile r_L as given; planned public construction uses r_T.

To verify optimality over the entire effort interval, hold the computed rival action and continuation value fixed and write the one-date objective, including current research costs, as $N(r)/D(r)$ plus a constant. The denominator has the form

$$D(r) = (1 + H + tr) \prod_{j:d_j=0} \{1 + Z_j(0) - \Lambda_{ji}\varepsilon_T r\} > 0.$$

Here H is the rival's current effective research intensity and t is the deviator's research productivity in the inherited feedback state. The polynomial degrees satisfy $\deg D \leq 3$ and $\deg N \leq 5$. The second derivative has numerator

$$N''D^2 - ND''D - 2N'DD' + 2N(D')^2 \tag{K.2}$$

and denominator D^3. Its Bernstein coefficients on $[0, .6]$ are evaluated at every state and laboratory for each capacity plan. Negative coefficients verify strict concavity over the entire feasible effort interval. The coefficient test is evaluated at the computed continuation values. The coupled nonlinear system is solved in floating-point arithmetic; the results are numerical solutions at the sampled assignments.

All 99 operating calculations converge. Among the 96 candidates passing the institutional checks below, the largest first-order residual is below 1.6×10^{-12} and the smallest computed uniform concavity bound exceeds .424. For a scalar objective with curvature at most $-m < 0$ and marginal residual ϵ, its possible improvement is bounded by $\epsilon^2/(2m)$, apart from the numerical continuation-value error. Among these 96 candidates, low- and high-capacity effort ranges from approximately .01088 to .1360, well below the feasible upper bound of .6.

K.3 Private rejection, fiscal funding and all waiting choices

The corporate envelope permits fractional assignments of infrastructure receipts, baseline ordinary profits, incremental ordinary profits, and the domestic laboratory's nonvoting claims. For a private construction deviation, its current operating kernel is M_L, while the installed continuation is that of H. The maximum return is obtained by summing the positive return components of those claims, state by state. With $G_k = (I - M_k)^{-1}\mathbf{1}$, these components are

$$b_R = M_L(R_H G_H - R_L G_L), \qquad b_0 = \Pi_L M_L(G_H - G_L),$$
$$b_\Delta = (\Pi_H - \Pi_L) M_L G_H, \qquad b_{\text{lab}} = M_L(V^R_{1H} - V^R_{1L}).$$

Thus $\overline{B}(x) = [b_R(x)]_+ + [b_0(x)]_+ + [b_\Delta(x)]_+ + [b_{\text{lab}}(x)]_+$. The smallest computed margin $J - \overline{B}$ among accepted candidates exceeds 17.41. Private rejection therefore holds with unrestricted construction funding.

Public current construction instead uses M_T. Let u_T and u_H be national operating-income vectors before engineering and collection losses, and let $U_H = (I - M_H)^{-1}u_H$ be the gross installed national value. Put

$$\begin{aligned} h &= M_T(I - M_H)^{-1}\mathbf{1}, \\ n^*(x,w) &= \frac{[J - w - R_L - h(x)R_H]_+}{1 + h(x)}, \\ C(x,w) &= (1 + h(x))\phi(n^*(x,w)), \\ V_B(x) &= u_T(x) - J + (M_T U_H)(x), \qquad C_{\max} = \max_x C(x,0). \end{aligned} \tag{K.3}$$

Country 1 finances the project and bears its collection losses; country 2's national account includes only its own operating income. The government's value under immediate construction is $V_B(x) - C(x,w)$. Funding satisfies the dated identity $w + R_L + n^* + h(R_H + n^*) = J$ whenever taxes are needed. The net tax cap has slack at every accepted point; all initial zero-wealth tax requirements are strictly positive.

Since $C(x,w)$ is nonnegative and decreasing in w, $C_{\max}$ bounds the collection loss over the entire interval $w \in [0, J]$. Following any one-date waiting, saving, tax or refinancing deviation, the candidate continuation value is bounded above by the gross construction value V_B, while current fiscal resource costs are nonnegative. Therefore

$$\min_x \{V_B(x) - u_T(x) - (M_T V_B)(x)\} - C_{\max} > 0 \tag{K.4}$$

rules out every open-access waiting financial action, independently of the wealth it produces. The positive-construction test uses the continued-low reference $(I - M_T)^{-1}u_T$ and the same cost bound. The inequality is uniform over the entire stated financial state set.

K.4 Access, joint policy deviations and inherited obligations

Access is evaluated separately on the private low-capacity path, the installed high-capacity path, and at current public construction. If country 1 owns the frontier, it can change access and investment together. Let u_T^c, M_T^c denote current income and the transition under restricted foreign commercial

use, holding already submitted research fixed. The following sufficient inequalities compare actual open construction with restricted construction and restricted waiting:

$$\begin{aligned} u_T - u_T^c + (M_T - M_T^c)U_H > C_{\max}, \\ V_B - u_T^c - M_T^c V_B > C_{\max}. \end{aligned} \tag{K.5}$$

They hold state by state whenever country 1 controls access. The restricted alternatives receive zero fiscal resource cost in these upper bounds. Claims and taxes are domestic transfers, and every feasible funding arrangement has nonnegative collection losses. The zero-cost bound also covers alternative feasible financing arrangements. Contracted proceeds and inherited payment obligations remain fixed under the within-period timing.

At installed capacity, every inherited normalized tax promise lies in $[0, 1]$. The remaining candidate collection loss is bounded by $\phi(1) \max_x G_H(x)$. If restriction changes the operating kernel, the gross access gain is required to exceed this entire bound. When both feedback sets are already validated, the kernel is structurally unchanged by current access, and lost export royalties establish strict dominance directly. This equality follows directly from the transition law. The foreign government's access comparisons use its own gross national income, while country 1 services the assigned domestic tax claims.

Previously assigned obligations remain fixed after issue. The candidate constant normalized tax stream is optimal under the verified open operating kernel by Proposition 11; the preceding bounds exclude joint deviations that change the kernel and financing. At a later low-capacity state, the same waiting and construction inequalities apply. The inequalities consequently verify continual decisions over all inherited financial positions in the stated institution.

K.5 Resource feasibility and results

For every feasible research choice, inference is at least .9994 in the smaller capacity. The largest research labor demand is .9012 per laboratory, leaving positive background employment in both countries. A conservative domestic cash-availability check permits simultaneously a deposit repayment of J, gross claim purchases of $[\beta a/(1-\beta a)](R_H+1)$, the full assigned tax call $1+\phi(1)$, and the largest laboratory funding call. The gross claim bound follows from the maximum quality-growth factor and covers issuance beyond the immediate funding gap. It leaves consumption resources above 906 in country 1 and 99 in country 2. It therefore covers $w \in [0, J]$, inherited $n \in [0, 1]$, and all feasible research deviations. These bounds deliberately omit positive ordinary profits and model royalties. Since $\beta a = .945 < 1$, bounded normalized deposits and claims satisfy the no-Ponzi condition and all continuation values are finite.

Table 8. Labor-income boundaries for the sufficient investment conditions

Feedback assignment σ	$L_1^\dagger$	$100J/L_1^\dagger$
.35	479.115	6.262
.80	519.720	5.772
.95	773.826	3.877

The last column expresses the engineering cost as a percentage of the domestic labor-income flow at the boundary. All values use the illustration's hypothetical primitives and are rounded for display. Financial states remain $w \in [0, J]$ and inherited normalized tax obligations remain in $[0, 1]$.

The listed sufficient checks pass at 96 of the 99 sampled values. At $\sigma = .01, .02, .99$, the uniform waiting bounds are respectively $-.3017$, $-.1114$ and $-.2781$, leaving those three operating solutions unclassified by the sufficient conditions. At the 34 tested values $\sigma \in \{.59, .60, \ldots, .92\}$, both domestic leadership shares fall under construction, while the domestic innovation rate, research in every state, national welfare and world growth increase. The smallest accepted uniform waiting margin exceeds .0043, the smallest joint construction-and-access margin exceeds .0407, and the smallest access margin exceeds .0426. The reported margins are evaluated at the numerical roots used in Figure 2.

K.6 The labor-income base and the timing of construction

The national gain depends on the labor-income base that benefits from quality growth. To measure this dependence, vary L_1 over $[100, 1000]$ at each of the three representative feedback assignments, retaining $L_2 = 100$ and all other primitives. On the open-access operating branch, normalized wages and research costs are unchanged, so the laboratory policies and transition matrices remain fixed. National values and the state-level policy margins are affine in L_1; project revenue, the tax requirement, and collection losses are unchanged.

Table 8 reports the numerical lower boundary $L_1^\dagger$ at which the stated sufficient institutional and resource conditions hold weakly. Each boundary is obtained from the affine state-level margins, with the constant margins checked separately. The uniform waiting condition (K.4) binds in all three cases. The remaining policy and resource inequalities have strict slack, and all checks pass strictly for $L_1 > L_1^\dagger$ within the indicated interval. These are boundaries of the sufficient tests on this operating branch. They measure how much labor income is needed for these tests to support immediate public construction.

The benchmark $L_1 = 1000$ lies above all three boundaries. Even at the boundaries, the conservative resident-resource calculation leaves more than 385 units in country 1 and 99 in country 2, and background employment remains positive for every feasible research choice. The binding

condition concerns the benefit of accelerating construction: the labor-income gain must cover both the project's resource costs and the opportunity to postpone them. This explains why the sufficient income requirement depends on the transition induced by the feedback assignment, as well as on the project's lifetime return.

K.7 Welfare decomposition and finite-horizon accounting

All welfare and transition calculations begin at domestic frontier ownership, unvalidated feedback and zero liquid wealth, with $Q_0 = 1$. For $k \in \{L, H, T\}$, write

$$v_k^{\text{corp}} = R_k + \Pi_k + \pi_{1k}^R, \qquad R_T = R_L, \quad \Pi_T = \Pi_L,$$

where π_{1k}^R is domestic laboratory income net of input and research costs under the relevant action profile. The corporate-income component in (38) is

$$\Delta V_1^{\text{corp}} = v_T^{\text{corp}} + M_T(I - M_H)^{-1} v_H^{\text{corp}} - (I - M_L)^{-1} v_L^{\text{corp}}. \tag{K.6}$$

The labor component uses $\mathbf{1} + M_T G_H - G_L$. Engineering expenditure occurs only at construction. With $n_0 = n^*(x_0, 0)$, the fiscal component is $[1 + (M_T G_H)(x_0)]\phi(n_0)$. In particular, the initially contracted n_0 remains fixed across subsequent realized states. Summing these components reproduces the difference between the two national value functions; the largest accounting discrepancy over all assignments and initial operating states is below 8.3×10^{-12}.

For finite horizons, let e_{x_0} be the unit row vector for the initial state. Physical state probabilities are

$$p_0^P = p_0^G = e_{x_0}, \qquad p_t^P = e_{x_0} P_L^t, \qquad p_t^G = e_{x_0} P_T P_H^{t-1} \quad (t \geq 1).$$

Frontier probabilities are $p_T^R I_1$. Cumulative expected domestic inventions are

$$N_1^P(T) = \sum_{t=0}^{T-1} p_t^P q_{1L}, \qquad N_1^G(T) = q_{1T}(x_0) + \sum_{t=1}^{T-1} p_t^G q_{1H}.$$

The discounted, quality-weighted rows are $z_t^P = e_{x_0} M_L^t$ and, for $t \geq 1$, $z_t^G = e_{x_0} M_T M_H^{t-1}$, with both date-zero rows equal to e_{x_0}. If $u_k = L_1 + v_k^{\text{corp}}$ denotes gross national income, actual discounted consumption over the first T dates is

$$W_1^P(T) = \sum_{t=0}^{T-1} z_t^P u_L,$$

$$W_1^G(T) = u_T(x_0) - J - \phi(n_0) + \sum_{t=1}^{T-1} z_t^G \{u_H - \phi(n_0)\mathbf{1}\}. \tag{K.7}$$

The matrices M_k already incorporate discounting, so these sums need no additional factor of β^t. Expected quality is obtained by removing discounting from the same rows. At $\sigma = .80$, the initial assigned stream is $n_0 = .2611173$.

The stationary covariance decomposition uses the invariant probability π_k of P_k separately at each capacity. The equality $\rho_1 - \theta_1 = \text{Cov}_{\pi_k}(q, I_1)/\bar{q}$ holds to within 9.3×10^{-16} over the 99 assignments. As a separate accounting check, summing 1,000 discounted dates under the actual T-then-H path and the permanent-L path reproduces the infinite-horizon welfare differences to within 2.9×10^{-11}. The bound $\beta a = .945$ controls the remaining discounted tail.