Information, Corporate Financing and Endogenous Dispersion and Volatility in Stock Returns*

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Abstract

This paper provides an unified explanation for the cyclical patterns of financial conditions, firm financing, and dispersion and volatility in stock returns. The model incorporates both ex-post and ex-ante monitoring, with the latter enabling mechanisms that credibly compels borrowers to reveal true investment outcomes, thereby reducing information asymmetry. As borrowers' financial conditions deteriorate, optimal financial contracts that minimize bankruptcy costs increasingly rely on ex-ante monitoring, making repayments contingent on disclosed information. This results in more equity-like contracts, with lenders assuming more idiosyncratic and aggregate risks, indicating increases in cross-sectional dispersion and market volatility in the returns of outside equity. Also, the implementation of optimal contract calls for more intensive use of equity financing, indicating that firms tend to substitute equity for debt financing in downturns.

Keywords: Asymmetric information, Dispersion, Volatility, Costly state verification, Financial contracts

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1 Introduction

Many studies have investigated the crucial role of asymmetric information in the amplification of adverse shocks, such as Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999)). In their models, there is typically no flexibility in firms' financing arrangement as only debt financing is allowed. Thus, firms cannot avoid the tightening of the information friction associated with debt financing during economic downturns. In this paper, we extend the conventional theories by considering the endogenous changes in firms' financing choices over business cycles to provide a unified explanation of some well-known stylized facts.

Empirical research has revealed two key facts about firms' financing patterns and stock market over business cycles:

Fact 1 (Financial structure) *Firms tend to finance with debt in booms and rely more on equity financing during recessions (e.g., Covas and Den Haan (2011), Jermann and Quadrini (2012) and Begenau and Salomao (2019)).*

Fact 2 (Dispersion in stock returns) *The time-series volatility of market returns and the cross-sectional dispersion of firm-level stock returns are countercyclical (e.g., Campbell and Lettau (1999) and Campbell, Lettau, Malkiel, and Xu (2001)).*

In this paper, we provide an unified explanation for the joint phenomena. Our model is a financial contracting framework that features costly state verification (CSV henceforth) of Townsend (1979). However, we deviate from the conventional models by allowing for both 'ex-ante monitoring' and 'ex-post monitoring'. Ex-post monitoring occurs when the borrower cannot repay the debt (i.e., costly bankruptcy), as in Townsend (1979). Ex-ante monitoring refers to setting up a mechanism in advance to credibly force the borrower to reveal the true investment outcome to the lender.[1] As pointed out by Myers and Majluf (1984), information asymmetry increases firms' cost to make public equity or debt offers. Thus, firms may voluntarily disclose information when they have strong incentives to mitigate the problems of information asymmetry and reduce the cost of external financing (Healy and Palepu (2001)).[2]

In the model, the borrower utilizes combinations of these two monitoring technologies. Although ex-ante monitoring is also costly, it enables financial contract

[1]For example, the lenders may require the firm to hire an independent auditor as a condition of lending.

[2]See Healy and Palepu (2001) for a summary of researches on the incentives and the consequences of information disclosure.

to be contingent upon the information it reveals, mitigating information asymmetry and reducing the expected bankruptcy (ex-post monitoring) cost. When ex-ante monitoring is not used at all, the optimal contract is standard debt as in Townsend (1979). When the borrower's investment is under fully ex-ante monitoring, information asymmetry is entirely eliminated and the optimal contract is standard equity, with repayment being perfectly correlated with investment outcomes. When both monitoring technologies are employed, the optimal contract is between a standard debt and standard equity.

When a borrower's financial condition worsens after an adverse shock, the likelihood of bankruptcy and the expected cost of ex-post monitoring increase. In such situations, the optimal contract calls for more intensive use of ex-ante monitoring. Consequently, the financial contract becomes more equity-like, with lenders assuming more aggregate and idiosyncratic risks. This leads to an increase in the cross-sectional dispersion of lenders' returns from the contracts. Also, firms' productivity declines because more resources are spent to set up the ex-ante monitoring.[3]

This mechanism differs from risk-sharing, since it is not about smoothing consumption between borrowers and lenders. Instead, in the model both parties are assumed to be risk-neutral. However, it is optimal to distribute more risks to the lender for a borrower with worsened financial conditions, as it helps reduce bankruptcy (ex-post monitoring) costs. As a result, it is the monitoring technology and bankruptcy cost minimization, rather than risk-sharing, that determines risk distribution between borrowers and lenders. This could be another implication of the costly state verification framework, which is less explored.

The theoretical results of our model can be interpreted in two ways. First, the financial contract can be interpreted as (outside) equity, with the level of information disclosure depending on the intensity of ex-ante monitoring. When an adverse shock worsens borrowers' financial conditions, ex-ante monitoring is used more intensively to reduce bankruptcy (ex-post monitoring) costs, making the optimal contract more equity-like. As a result, more aggregate and idiosyncratic risks are distributed to the lenders, leading to increases in both the time-series volatility of market return and the cross-sectional dispersion of firm-level returns.

Moreover, following an adverse shock the cross-sectional dispersion of borrowers' funding costs and expected productivities also increase. This is because non-financially constrained borrowers make the same financing and production decisions regardless of their wealth levels, while financially constrained borrowers make different decisions based on their wealth levels. A negative shock that shifts the wealth distribution to the left increases the fraction of financially con-

[3] Another possible interpretation is that (ex-ante) monitoring places some restrictions on production activities, leading to lower productivity.

strained borrowers, leading to greater heterogeneity in borrowers' financing and production decisions. Consequently, borrowers' financing decisions and expected productivities become more dispersed. This prediction of our model is inline with Gilchrist, Sim, and Zakrajšek (2013) who documented a sharp increase in the dispersion of firms' borrowing costs after 2007, reflecting the increased heterogeneity in firms' degrees of financial constraint after the crisis.

To quantitatively assess the proposed mechanism, we show that the calibrated model, subjected to a first-moment shock, generates cyclical patterns of dispersion in asset returns, market volatility and financial conditions that are inline with observed stock market phenomena: (1) cross-sectional dispersion and market volatility are countercyclical and highly correlated; (2) cross-sectional dispersion and market volatility are positively correlated with financial condition indicators (e.g.,Campbell and Lettau (1999) and Bloom (2009)). The model-generated correlation coefficients between these measures align quantitatively with empirical counterparts. A numerical exercise shows that after a first-moment shock with no persistence, increases in dispersion and market volatility are significant and persistent. This persistence arises because cross-sectional dispersion and market volatility in asset returns are largely determined by the distribution of borrowers' financial conditions, which takes time to recover.

Another interpretation of our theoretical results would be to consider the financial contract as a mixture of standard debt and equity, with the non-contingent payments being attributed to debt holders and payments contingent on the observed performance being attributed to equity holders. Thus, when ex-ante monitoring is used more intensively (following a negative shock that worsens borrowers' financial conditions), more information is disclosed and thus a larger fraction of payments to the lenders are made contingent on the observed performance. This implies that borrowers with worsened financial conditions substitute equity financing for debt financing, which is inline with observed cyclical patterns of firm financing (e.g., Covas and Den Haan (2011), Jermann and Quadrini (2012) and Begenau and Salomao (2019)).

Related Literature. Measures of stock market volatility, such as the cross-sectional dispersion of individual stock returns and the time-series volatility of market returns, tend to rise during economic downturns. The significant countercyclical movements of these two measures are often used as evidence supporting the notion that uncertainty shock – exogenous increase in uncertainty – negatively impact aggregate economic activities and lead to recessions (e.g., Bloom (2009), Arellano, Bai, and Kehoe (2010), Gilchrist, Sim, and Zakrajšek (2014), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)).[4] This body of research primarily focuses on how exogenous increases in uncertainties, propagated by various frictions, discourage economic activities. However, Cesa-Bianchi, Pesaran, and Rebucci (2020) investigate a multicountry panel and find that when conditioning world growth shocks, the correlation between volatility and economic growth become quantitatively much smaller and statistically insignificant, which implies that the explanatory power attributed to volatility shocks might be due to omitted common factors.[5]

Our study complements the literature by providing an alternative explanation for the observed countercyclical movements of dispersion and market volatility. We propose a mechanism through which a negative first-moment shock that worsens financial conditions leads to increases in both the cross-sectional dispersion and market volatility of stock returns.

In particular, our model predicts a positive correlation between cross-sectional dispersion and market volatility in asset returns. This significant empirical pattern is well-documented but may not have been clearly explained by existing studies. An increase in idiosyncratic risks, i.e., risks that can be reduced or eliminated through diversification, can indeed increase cross-sectional dispersion but not necessarily the time-series volatility of market average. If there exists a linkage between aggregate and idiosyncratic risks, what is it, and how does it translate into the observed correlation between cross-sectional dispersion and aggregate volatility? This paper examines this phenomenon from a different perspective: Since the amounts of idiosyncratic and aggregate risks distributed to lenders are both determined by borrowers' financial conditions, cross-sectional dispersion and market volatility tend to move together.

Our paper is also closely related to the literature on the dynamic pattern of firm financing over the business cycle, such as Hackbarth, Miao, and Morellec (2006), Covas and Den Haan (2011, 2012), Jermann and Quadrini (2012) and Begenau and Salomao (2019). Also, Bolton and Freixas (2000) studies the cross-sectional pattern of firm financing. In contrast with their models, our model emphasizes

[4] Bloom (2009) estimated a series of vector autoregressions (VARs) and demonstrated that an uncertainty shock, represented by a shock to the market volatility index, leads to a rapid fall of industrial production and employment. Similarly, Gilchrist, Sim, and Zakrajšek (2014) conducted a VAR analysis showing that an uncertainty shock, indicated by a shock to stock return volatility, results in a widening spread between interest rates on corporate and Treasury bonds, and decreases in output and investment. Bachmann, Elstner, and Sims (2013) measures uncertainty with forecast error disagreement.

[5] There is also a substantial body of literature on asset pricing highlights the significance of countercyclical volatility in comprehending stock market returns (e.g., Bansal, Kiku, Shaliastovich, and Yaron (2014), Campbell, Giglio, Polk, and Turley (2018) and Ai, Li, and Yang (2020).

the important role of information channel in determining firms' financing pattern over business cycles. In our model, the optimal contract that requires repayment to be contingent on the information it reveals can be implemented using a mixture of debt and equity financing. As the optimal contract becomes more equity-like in economic downturns, more equity financing is needed in the implementation of the optimal contract.

Our paper is also related to studies on endogenous uncertainty. For example, Tian (2015) studies firms' risk-taking, predicting that small firms tend to take more risks, and since the fraction of risky firms rises during recessions, leading to countercyclical productivity dispersion. Bachmann and Moscarini (2011) examine firms' price-setting behavior when information about demand elasticity is imperfect, predicting that economic downturns are opportune times for firms to experiment with pricing to gain information on market power, increasing price dispersion during recessions. In their theories, total risks in the economy rise during downturns as firms create more idiosyncratic risks when a negative shock shifts them closer to the exit point where their value functions are locally convex. However, this paper focuses on how investment risks are distributed among different agent groups, while total risks within the economy remain unchanged.

The modeling in our paper is closely related to Boyd and Smith (1999), which studies optimal capital structure in a similar CSV framework.[6] However, there are some important differences between the two papers. Their paper focuses on how optimal capital structure changes with monitoring costs, while our paper introduces borrowers' net worth and focuses on how optimal contracts change with borrowers' financial conditions. Also, our paper introduces heterogeneity in borrowers' wealth, where decisions of financially constrained borrowers are based on their wealth levels.

The CSV framework has been a workhorse for macroeconomists. There has been a growing literature on the 'financial accelerator', embedding the costly state verification problem into DSGE models, such as Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999). These papers primarily focus on how information asymmetry amplifies real shocks and leads to fluctuations in investment, employment and output. Although this paper also takes the CSV problem as a starting point, it focuses on the role of financial friction in determining the dynamic patters of corporate financing over business cycles.

The rest of this paper is organized as follows. Section 2 describes the model.

[6]Actually, papers on security design and capital structure are closely related to each other. The former views the optimal contract as an optimal mechanism for mitigating frictions, while the later takes standard debt and equity contracts as given and focuses on the optimal mix of them (Allen and Winton (1995)).

Section 3 applies the model to account for the observed countercyclical fluctuations in cross-sectional dispersion and market volatility in stock returns (Fact 2). Section 4 applies the model to explain the cyclical patters of firms financial structure (Fact 1). Section 5 concludes.

2 The Model

We propose a tractable dynamic model with costly state verification.

2.1 Environment

Time is discrete and the horizon is infinite, i.e., $t = 0, 1, 2, ...$ There is only one goods, which can be either consumed or invested.

There is a storage technology that yields R units of goods in the next period for each unit goods stored in the current period, where R is exogenously given.

There are two groups of agents: borrowers and lenders, with the mass of each group being normalized to one. Borrowers are indexed by $j \in [0, 1]$. However, for convenience, we will temporarily drop the borrower's index j. Lenders are risk neutral, and can either lend to the borrowers or invest in the storage technology that has a gross return of R.

In period t, borrower j starts with w_{jt} units of goods. The wealth distribution is denoted by the cumulative density function, $\Phi(w)$. Borrowers discount future consumption at rate β. Borrowers are risk neutral, and the life-time utility of a borrower is given as $\mathbb{E} \sum_{t=1}^{\infty} \beta^t c_t$, where c_t is the consumption in periods t.

In each period, the borrower is endowed with an investment opportunity, called 'project'. To start a project, the borrower has to invest *one* unit of goods. The part of funds invested by the borrower himself is called 'net worth', denoted by n. The rest of the funds invested, $1 - n$, is borrowed from the lenders.

Ex-ante and ex-post Monitoring. Ex-post monitoring happens only when the promised payment is not made and the lenders take all the residual (i.e., when bankruptcy happens), as in Townsend (1979). Ex-ante monitoring refers to setting up a mechanism in advance that can credibly force the borrower to disclose the actual investment revenue to the lender.

We denote by α the fraction of the borrower's investment under the ex-ante monitoring. To set up the ex-ante monitoring mechanism, a fraction $1 - \rho$ of the invested funds is spent before production takes place. With the ex-ante monitoring, the borrower's total investment revenue consists of a publicly observed part and a privately observed part:

$$\underbrace{\alpha \cdot e^{z} \cdot (\rho R_k)}_{\text{publicly observed}} + \underbrace{(1-\alpha) \cdot e^{u} \cdot R_k}_{\text{privately observed}}.$$

Here, R_k is the aggregate investment return, which is observed by the public, and (z, u) are two idiosyncratic shocks drawn from the same distribution, with $\mathbb{E}(e^z) = \mathbb{E}(e^u) = 1$. Shock z is observed by both the borrower and lender, while u is privately observed by the borrower. Since a fraction $1-\rho$ of invested funds is used to set up the ex-ante monitoring, the expected net return of investment under ex-ante monitoring is ρR_k.[7]

For the fraction of investment not under the ex-ante monitoring, the lender can pay an ex-post monitoring cost to observe the realized revenue, which equals a fraction μ of the unobserved revenue, i.e. $\mu \cdot e^u R_k$.

Obviously, when ex-ante monitoring is more costly than ex-post monitoring, i.e., $1-\rho > \mu$, it is never optimal to set up the ex-ante monitoring mechanism. This is because the cost of ex-ante monitoring is incurred with certainty, while the cost of ex-post monitoring is incurred only in the event of bankruptcy. In this case, the borrower would optimally choose $\alpha = 0$, and the optimal contract is standard debt as in Townsend (1979).

When the ex-ante monitoring is free, i.e. $1-\rho = 0$, the borrower always set up full ex-ante monitoring, which completely eliminated information asymmetry. In this case, $\alpha = 1$, and the optimal contract is standard equity.

When the ex-ante monitoring cost lies between 0 and μ, the optimal contract should be something between standard debt and standard equity. To ensure that there is a trade-off between the two monitoring technologies, we assume that $0 < 1-\rho < \mu$.

Assumption 1 *The cost of ex-ante monitoring is between zero and the cost of ex-post monitoring, i.e.,* $0 < 1-\rho < \mu$.

Furthermore, each of (z, u) consists of two components: an aggregate component and idiosyncratic components:

$$z = s + \varepsilon_z \qquad \text{and} \qquad u = s + \varepsilon_u,$$

where s denotes the aggregate component and ε_z and ε_u are the idiosyncratic components, which are i.i.d. across borrowers and over time. The idiosyncratic

[7] For example, lenders may require the borrower to hire an independent auditor, who can enhance the credibility of information the borrower discloses, as a condition of lending.

components (ε_z and ε_u) and the aggregate component s are independent. In addition, for the same borrower, ε_z and ε_u are independent of each other. Finally, we assume $s \sim \mathcal{N}(-\sigma_s^2/2, \sigma_s^2)$ and $\varepsilon_z, \varepsilon_u \sim \mathcal{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$. Thus, $\mathbb{E}(e^z) = \mathbb{E}(e^u) = 1$.

Since z and u share the same aggregate component, they are not independent of each other. In fact, z and u are independent only conditional on s. Notice that the joint normal distribution of s and z is given by:

$$\begin{bmatrix} s \\ z \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} -\sigma_s^2/2 \\ -(\sigma_s^2 + \sigma_\varepsilon^2)/2 \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & \sigma_s^2 \\ \sigma_s^2 & \sigma_s^2 + \sigma_\varepsilon^2 \end{bmatrix} \right)$$

We assume that even if monitoring occurs, only individual investment returns, z and u, can be observed, and borrowers and lenders cannot directly observe the realized aggregate state s before the repayment process concludes. The idiosyncratic components obscure the aggregate component. Thus, absent of ex-post monitoring, financial contract can be made only contingent on state z that is observable to both the borrower and lender.[8] However, the realized value of z does provide some information on s and u. Note that conditional on z, the conditional distribution of s is given by:

$$s|z \sim \mathcal{N}(\kappa z, \tilde{\sigma}_s^2),$$

where $\kappa \equiv \sigma_s^2/(\sigma_s^2 + \sigma_\varepsilon^2)$ and $\tilde{\sigma}_s^2 \equiv \sigma_s^2\sigma_\varepsilon^2/(\sigma_s^2 + \sigma_\varepsilon^2)$. Also, the distribution of u conditional on z is

$$u|z \sim \mathcal{N}(\kappa z + \bar{\varepsilon}, \tilde{\sigma}_u^2).$$

where $\tilde{\sigma}_u^2 \equiv \tilde{\sigma}_s^2 + \sigma_\varepsilon^2$ and $\bar{\varepsilon} \equiv -\sigma_\varepsilon^2/2$. We denote by $g(z)$ the density functions of z, and denote by $f(u|z)$ and $F(u|z)$ the density and cumulative density functions of u conditional on z.

2.2 The borrower's optimization problem

The borrower's optimization problem is solved in two steps. In Section 2.2.1, we consider the optimal one-period financial contract singed with the lenders. In Section 2.2.2, we consider the inter-temporal optimization problem of the borrower.

2.2.1 Financial Contract

To finance their investments, borrowers not only borrow from lenders but also invest their own wealth into the project, referred to as 'net worth'. As in the

[8] In the real world, stock returns are contingent on the observed performances of firms, but not the state of aggregate economy.

conventional CSV framework, the optimal contract maximizes the borrower's expected profits while ensuring the lender receives an expected return not less than their opportunity cost. The optimal contract minimizes expected (ex-ante and ex-post) monitoring costs.

Given the net worth n, the borrower borrows $1-n$ from the lender and signs a financial contract, denoted by $\mathcal{A}$. The financial contract specifies: (1) monitoring scheme (the value of α); (2) repayment rules. Since z is observable to both the borrower and the lender, the repayment rules can always be made contingent on z. However, the repayments can be made contingent on u only under the ex-post monitoring. Let $D(z)$ denote the ex-post monitoring region and $D^c(z)$ the non-monitoring region, given the realized z, with $D(z) \cup D^c(z) = (-\infty, \infty)$. Let $x(z)$ denote the promised repayment to the lender in the non-monitoring region and $r(z, u)$ be the repayment when ex-post monitoring takes place. Thus, a financial contract is a set of contract terms that specify monitoring scheme and repayment rules, i.e. $\mathcal{A} = \{\alpha, x(z), r(z, u), D(z)\}$.

In addition, a financial contract should satisfy the following constraints:

Limited liability. Neither the borrower nor the lender can get a negative return:

$$0 \leq x(z) \leq [\alpha\rho e^z + (1-\alpha)e^u]R_k \quad and \quad 0 \leq r(z,u) \leq [\alpha\rho e^z + (1-\alpha)e^u]R_k. \tag{1}$$

Incentive compatibility. The financial contract should also be incentive compatible, such that the borrower never has incentive to cheat on the realization of u when a monitoring state occurs. It requires that the repayment in the monitoring region is no more than that in the non-monitoring region:

$$r(z,u) \leq x(z) \quad for\ all\ u \in D(z). \tag{2}$$

Promised return. We assume that the borrower has the whole bargaining power. Thus, the lender gets an expected return that equals the return of the storage technology, R:

$$\int_{-\infty}^{\infty} \left\{ \int_{u \in D^c(z)} x(z) f(u|z) du + \int_{u \in D(z)} [r(z,u) - \mu(1-\alpha)e^u R_k] f(u|z) du \right\} g(z) dz = (1-n)R. \tag{3}$$

No short sale. The fraction of investment under ex-ante monitoring, α, should be between zero and one:

$$0 \leq \alpha \leq 1. \tag{4}$$

Let $\Omega(n)$ denote the set of financial contracts that satisfy constraints (1)-(4). Given the net worth n, the optimal contract is chosen from $\Omega(n)$ to maximize the borrower's expected profits.

Problem 1 (Optimal contract) *Given net worth n, the optimal contract solves*

$$\pi(n) = \max_{\mathcal{A}\in\Omega(n)} [\alpha\rho + (1-\alpha)]R_k - (1-n)R$$
$$-\mu(1-\alpha)R_k \int_{-\infty}^{\infty} \left(\int_{u\in D(z)} e^u f(u|z)du \right) g(z)dz.$$

In the above expression for $\pi(n)$, the first term is the expected investment revenue, the second is the promised payment to the lender, the third term is the dead weight loss of ex-post monitoring.

If the expected value of observable revenue, $\alpha\rho R_k$, is large enough such that $\alpha\rho R_k \geq (1-n)R$ holds, there is no need for ex-post monitoring. In this case, the borrower can simply assure the lender with a fraction θ of the observable revenue $\alpha e^z(\rho R_k)$, such that, $(1-n)R = \theta\alpha\rho R_k$. However, as will be shown below, this is never optimal because the marginal cost of ex-post monitoring is nearly zero when ex-post monitoring never occurs.

If the expected value of observed revenue is smaller than the promised value to the lender, $\alpha\rho R_k < (1-n)R$, then $x(z) \geq \alpha e^z(\rho R_k)$ should hold at least for some states of z. Note that $x(z)$ is the payment outside the monitoring region and $\alpha\rho e^z R_k$ is the observable revenue. Thus, the differential, $x(z) - \alpha\rho e^z R_k$, is from the unobserved revenue. It is helpful to define the following threshold of u:

$$\hat{u}(z) \equiv ln\left[\frac{x(z) - \alpha\rho e^z R_k}{(1-\alpha)R_k}\right]. \tag{5}$$

Thus, if $u < \hat{u}(z)$ the repayment $x(z)$ will be infeasible.

As can be seen, given the monitoring scheme α and the realized z, the problem is similar to a standard costly verification problem with only one unobserved state, u.

Lemma 1 *Suppose $(1-n)R > \alpha(\rho R_k)$. Then, the optimal contract features:*

1. *$A(z) = (-\infty, \hat{u}(z))$;*
2. *When $u \notin D(z)$, $x(z) \geq \alpha e^z(\rho R_k)$;*
3. *When $u \in D(z)$, $r(z,u) = \alpha e^z(\rho R_k) + (1-\alpha)e^u R_k$.*

Proof. *See Appendix A2.1.* ■

Lemma 1 indicates that ex-post monitoring never happens if it is feasible to repay $x(z)$. When repaying $x(z)$ is infeasible, ex-post monitoring happens and the lender takes away all the residual.

By using the results in Lemma 1, Problem 1 simplifies to

Problem 2 (Reduced) *The optimal contracting problem 1 can be simplified to*

$$\pi(n) = \max_{\alpha,\{\hat{u}(z)\}} [\alpha\rho + (1-\alpha)]R_k - (1-n)R - (1-\alpha)R_k \int_{-\infty}^{+\infty} \Psi(\hat{u}(z)|z)g(z)dz$$

subject to the promise-keeping constraint

$$\alpha(\rho R_k) + (1-\alpha)R_k \int_{-\infty}^{+\infty} [\Gamma(\hat{u}(z)|z) - \Psi(\hat{u}(z)|z)]\, g(z)dz = R(1-n),$$

where

$$\Gamma(\hat{u}(z)|z) \equiv e^{\hat{u}(z)}[1 - F(\hat{u}(z)|z)] + \int_{-\infty}^{\hat{u}(z)} e^{u} f(u|z)du,$$

$$\Psi(\hat{u}(z)|z) \equiv \mu \int_{-\infty}^{\hat{u}(z)} e^{u} f(u|z)du.$$

Notice that in the above expression, $\Gamma(\hat{u}(z)|z)$ is the *lender's share* of the unobserved revenue, $(1-\alpha)R_k$, and $\Psi(\hat{u}(z)|z)$ is the *dead weight loss* of monitoring.

The first order condition for $\hat{u}(z)$ is given by:

$$\frac{1}{1+\lambda} = \frac{\Gamma'(\hat{u}(z)|z) - \Psi'(\hat{u}(z)|z)}{\Gamma'(\hat{u}(z)|z)} \quad \textit{for all } z. \tag{6}$$

where λ is the Lagrangian multiplier associated with the promise-keeping constraint. Thus, $1/(1+\lambda)$ is interpreted as the *marginal rate of transformation*: to increase the expected repayment to the lender by one unit, it requires a $1+\lambda$ units decrease in the borrower's expected profit, where λ is the loss rate of transformation due to ex-post monitoring. Also, the shadow price of net worth is given by $(1+\lambda)R$, indicating that the inside funds is more valuable than the outside funds due to financial friction.

Notice that $\Gamma'(\hat{u}(z)|z) = e^{\hat{u}(z)}[1 - F(\hat{u}(z)|z)]$ and $\Psi'(\hat{u}(z)|z) = \mu e^{\hat{u}(z)} f(\hat{u}(z)|z)$, where $f(u|z)$ and $F(u|z)$ are the p.d.f. and c.d.f. of u conditional on z, and also that $u|z \sim \mathcal{N}(\kappa z + \bar{\varepsilon}, \tilde{\sigma}_u^2)$, indicating that $(u - \kappa z)|z \sim \mathcal{N}(\bar{\varepsilon}, \tilde{\sigma}_u^2)$, as mentioned in Section 2.1. Thus, condition (6) can be re-written as

$$\frac{1}{1+\lambda} = 1 - \frac{\mu \tilde{f}(\hat{u}(z) - \kappa z)}{1 - \tilde{F}(\hat{u}(z) - \kappa z)} \quad for\ all\ z,$$

where $\tilde{f}(\cdot)$ and $\tilde{F}(\cdot)$ denote the p.d.f. and c.d.f. of $\mathcal{N}(\bar{\varepsilon}, \tilde{\sigma}_u^2)$.[9] Since λ does not depend on z, thus $\hat{u}(z) - \kappa z$ are equal across state z.[10] This implies

$$\frac{1}{1+\lambda} = 1 - \frac{\mu \tilde{f}(\hat{u})}{1 - \tilde{F}(\hat{u})}. \tag{7}$$

Here, $\hat{u}$ is referred to as *the baseline threshold* of u. The borrower adjusts the threshold according to the realization of z, i.e., $\hat{u}(z) = \hat{u} + \kappa z$, in order to smooth the bankruptcy cost across state z.

Define $\bar{\Gamma}(\hat{u}) \equiv \int_{-\infty}^{\infty} \Gamma(\hat{u}(z)|z) g(z) dz$ and $\bar{\Psi}(\hat{u}) \equiv \int_{-\infty}^{\infty} \Psi(\hat{u}(z)|z) g(z) dz$. Thus, $\bar{\Gamma}(\hat{u})$ is the expected lender's share and $\bar{\Psi}(\hat{u})$ is the expected dead weight loss of ex-post monitoring. Thus, we have (remember that $\hat{u}(z) = \hat{u} + \kappa z$)

$$\bar{\Gamma}(\hat{u}) = e^{-\tilde{\sigma}_s^2/2} \left\{ e^{\hat{u}}[1 - \tilde{F}(\hat{u})] + \int_{-\infty}^{\hat{u}} e^{\nu} \tilde{f}(\nu) d\nu \right\},$$

$$\bar{\Psi}(\hat{u}) = \mu e^{-\tilde{\sigma}_s^2/2} \int_{-\infty}^{\hat{u}} e^{\nu} \tilde{f}(\nu) d\nu.$$

The first order condition for α is thus given by:

$$\rho = \frac{1}{1+\lambda}(1 - \bar{\Gamma}(\hat{u})) + (\bar{\Gamma}(\hat{u}) - \bar{\Psi}(\hat{u})). \tag{8}$$

The above condition states that the marginal return of investment under ex-ante monitoring equals that of investment absent of ex-ante monitoring. To see this, note first that the marginal return of investment under ex-ante monitoring is ρR_k, which is completely observable and thus pledgeable. This corresponds the left-hand-side of the above equation. Note also that the marginal return of investment absent of ex-ante monitoring is R_k, and only a fraction $\bar{\Gamma}(\hat{u}) - \bar{\Psi}(\hat{u}))$ of which is pledgeable (remember that $\bar{\Gamma}(\hat{u})$ is the share of expected revenue delivered to the lender and $\bar{\Psi}(\hat{u})$ is the dead weight loss due to ex-post monitoring). The remaining fraction, $1 - \bar{\Gamma}(\hat{u})$, is kept by the borrower, which is non-pledgeable and thus discounted by the *marginal transform rate*, $1/(1+\lambda)$. This condition suggests that the marginal costs of ex-ante and ex-post monitoring must equalize.

[9] Note that $F(u|z) = \tilde{F}(u - \kappa z)$ and $f(u|z) = \tilde{f}(u - \kappa z)$.

[10] Note that $\tilde{f}(\cdot)$ is monotonically decreasing over $(-\infty, 0)$.

Furthermore, notice that the value of λ and $\hat{u}$ are jointly determined by conditions (7) and (8), this implies that λ and $\hat{u}$ are just functions of parameters:

$$\lambda = \lambda(\rho, \mu, \sigma_s^2, \sigma_\varepsilon^2) \quad \textit{and} \quad \hat{u} = u(\rho, \mu, \sigma_s^2, \sigma_\varepsilon^2).$$

Then, the promise-keeping constraint in Problem 2 can be re-written as:

$$(1-n)R = \alpha(\rho R_k) + (1-\alpha)(\bar{\Gamma}(\hat{u}) - \bar{\Psi}(\hat{u}))R_k \tag{9}$$

By re-arranging the above condition, we have:

$$1 - \alpha = \frac{\rho R_k - (1-n)R}{\rho R_k - (\bar{\Gamma}(\hat{u}) - \bar{\Psi}(\hat{u}))R_k} \tag{10}$$

As can be seen, the fraction of investment absent of ex-ante monitoring, $1 - \alpha$, is linearly increasing in the net worth n. When the net worth is low, the borrower depends more on the ex-ante monitoring mechanism that helps mitigate the agency problem.

Assumption 2 *Parameter ρ satisfies $\rho R_k - R > 0$.*

Expression (10) also implies that α may hits its upper or lower bounds when n is small or large enough. Assumption 2 ensures that it not optimal to choose $\alpha = 1$ even if the borrower puts zero net worth in the project. Thus, the constraint $\alpha \le 1$ never binds.

However, when the net worth n is large enough, α may hit its lower bound 0. In fact, expression (10) indicates that there exists a critical value of net worth, $\bar{n}$, such that the constraint $\alpha \ge 0$ binds when $n > \bar{n}$, which is given by:

$$\bar{n} = 1 - [\bar{\Gamma}(\hat{u}) - \bar{\Psi}(\hat{u})]\frac{R_k}{R}. \tag{11}$$

By using the above results, we now characterize the optimal contract.

Proposition 1 (Optimal contract) *Suppose $n \in (0, \bar{n})$. Thus, the non-negative constraint (4) does not bind, and the optimal contract is specified by:*

1. *$D(z) = (-\infty, \hat{u} + \kappa z)$;*
2. *When $u \notin D(z)$, $x(z) = [\alpha\rho e^z + (1-\alpha)e^{\hat{u}+\kappa z}]R_k$;*
3. *When $u \in D(z)$, $r(z, u) = [\alpha\rho e^z + (1-\alpha)e^u]R_k$*

where λ and $\hat{u}$ are functions of parameters as implied by (7) and (8), and α is determined by condition (9).

Finally, by using all the result, the expected return of the project is given as

$$\pi(n) = \tilde{R}\left(n + \frac{\rho R_k - R}{R}\right), \tag{12}$$

where $\tilde{R} \equiv (1+\lambda)R$ (remember that λ being a function of parameters as shown before).

2.2.2 Inter-temporal optimization

In each period, given the current wealth w, the borrower invests n in his/her project and consumes $c = w - n$. Given the net worth n, the borrower signs an one-period financial contract with the lenders as in the simple model.

Remember that productivity shocks s, ε_z and ε_u are i.i.d. over time. Thus, the only endogenous aggregate state is the distribution of the borrowers' wealth, denoted by $\Phi(w)$. Thus, the borrower's optimization problem is as follows.

Problem 3 *The borrower solves*

$$V(w|\Phi) = \max_{n\in[0,w]} c + \beta\mathbb{E}[V(w'|\Phi')], \tag{13}$$

subject to

$$c = w - n \qquad \text{and} \qquad w' = \pi(n, z', u'|\Phi).$$

Since the borrower's expected investment revenue, $\pi(n) = \mathbb{E}\pi(n, z', u'|\Phi)$, given by equation (12), is linear in the net worth n. To ensure that the model has interior solutions, we made the following assumption.

Assumption 3 *Parameter β satisfies $\beta\tilde{R} = 1$.*

Under Assumption 3, the borrower is indifferent between consumption and investment, and thus the level of investment is undetermined. Thus, we further assume that the borrower always chooses to invest when he/she is indifferent between consumption and investment.

Assumption 4 *When the borrower is indifferent between consumption and investment, he/she chooses to invest.*

Under Assumptions 3 and 4, the optimal investment policy is quite simple. When the initial wealth w is less than the threshold $\bar{n}$, he/she invests all the funds in the project, i.e., $n = w$. When $w > \bar{n}$, the borrower invests $\bar{n}$ in the project and consumes $w - \bar{n}$. This is because one n exceeds $\bar{n}$, the marginal return of investment drops below $1/\beta$.

Proposition 2 (Optimal investment) *Given the borrower's initial wealth w:*

1. *If $w \leq \bar{n}$, the borrower invests all the funds in the project, i.e. $n = w$;*
2. *If $w > \bar{n}$, the borrower invests $\bar{n}$ in the project and consumes the rest, i.e. $n = \bar{n}$ and $c = w - \bar{n}$;*

where the threshold $\bar{n}$ is defined by (11).

Therefore, a borrower with $w > \bar{n}$ is referred to as a '*non-financially constrained*' borrower in the sense that his/her consumption is strictly positive.

We can show that the borrower's value is given as

$$V_t(w_t) = w_t + \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\rho \hat{e}^{z} \left(\frac{R_{k,t+\tau}}{R} \right) - 1 \right].$$

2.3 Equilibrium

In each period, given the borrowers' financing decisions, the aggregate investment, denoted by K, is defined as

$$K = \int_0^1 [\rho \alpha_j + (1 - \alpha_j)] dj.$$

Remember that setting up the ex-ante monitoring mechanism costs a fraction $1 - \rho$ of the invested funds. Thus, K is the gross investment (which is one) net of the cost of ex-ante monitoring.

To close the model, we assume that the aggregate investment return R_k decreases with the aggregate capital stock:

$$R_k = AK^{\theta - 1},$$

where $\theta \in (0,1)$ and A is a constant. One can think that there are goods producers who rent capital from the borrowers and R_k is the rental price. Then, the aggregate output is given by

$$Y = \int_0^1 \left[\rho e^{z_j}\alpha_j + e^{u_j}(1-\alpha_j)\right] R_k dj = e^s A K^\theta, \tag{14}$$

where as before e^s is the aggregate shock.[11].

The evolution of the distribution of borrowers' wealth is denoted by

$$\Phi(w') = \Theta(\Phi(w), s').$$

3 Cyclical dispersion and volatility

The countercyclical behavior of the cross-sectional dispersion and market volatility in stock returns is a well-documented phenomenon. Also, these metrics are closely linked to financial condition indicators. Campbell and Lettau (1999) and Campbell, Lettau, Malkiel, and Xu (2001) demonstrated that both the dispersion and market volatility of stock returns exhibit countercyclical patterns. Bloom (2009) found that the stock market volatility index, such as the VXO, is countercyclical and positively co-varies with the cross-sectional dispersion of firm stock returns and profits. Gilchrist, Sim, and Zakrajšek (2014) showed that stock return volatility is countercyclical and strongly correlated with financial condition indicators, such as credit spreads, at both the firm and aggregate levels.

As Figure 1 shows, during the crisis, the financial conditions is significantly worsened, and the cross-sectional dispersion and market volatility in stock returns rise sharply.

To account for these facts, we interpret the financial contract in our model as (outside) equity, and provide numerical exercises showing that a negative first-moment shock can lead to increases in cross-sectional dispersion and market volatility in stock returns through deteriorating financial conditions. The central idea is that when a borrower's financial condition deteriorates, the likelihood of bankruptcy (ex-post monitoring) rises. Consequently, ex-ante monitoring is used more intensively, which helps reduce information frictions. However, this also distributes more aggregate and idiosyncratic risks to the lenders, leading to increases in the cross-sectional dispersion and market volatility in stock returns.

[11]Note that $z_j = s + \varepsilon_{z,j}$ and $u_j = s + \varepsilon_{u,j}$ where the idiosyncratic components ($\varepsilon_{z,j}$ and $\varepsilon_{u,j}$) are i.i.d. across borrowers and do not depend on borrowers' monitoring scheme, α_j.

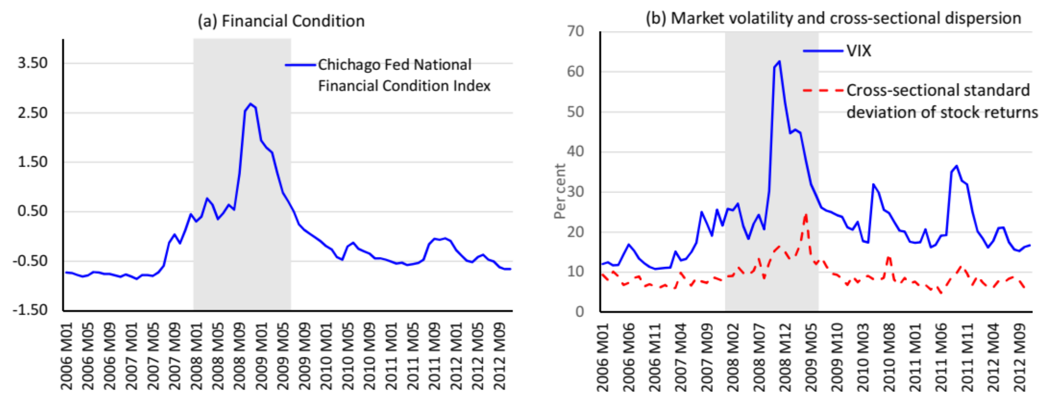

Figure 1: Dispersion, volatility and financial condition

Notes: Panel (a) presents the Chicago Fed National Financial Condition Index. Panel (b) shows the movements of cross-dispersion of stock returns and market volatility measured by the VIX index (a widely used measure of implied market volatility). The data consist of monthly stock returns from 2003 to 2012, using the same basket of selected stocks as in Bloom (2009). Shaded bars indicate official NBER recessions.

3.1 Measures for dispersion and volatility

Before analyzing how the cross sectional dispersion and the market volatility in asset returns change with financial conditions, it is helpful to formally define these measures.

Recall that bankruptcy (and ex-post monitoring) occurs only when the realized u is below the threshold $\hat{u}(z_j)$. Define $\mathcal{S} \equiv \{j \,|u_j > \hat{u}(z_j)\}$ to be the set of borrowers who have not gone bankrupt. Recall also that $\hat{u}(z_j) = \hat{u} + \kappa z_j$, where $\hat{u}$ is a function of parameters and common to all the borrowers (see Section 2.2.1). Thus, the set for solvent borrowers, $\mathcal{S}$, can be re-written as

$$\mathcal{S} = \left\{j \,|u_j > \hat{u} + \kappa z_j\right\}.$$

Thus, bankruptcy rate is given by

$$Bankruptcy\ rate \ = 1 - \int_{j \in \mathcal{S}} 1.$$

We can show that the bankruptcy rate depends negatively on the aggregate state s.[12]

[12]Recall that $z_j = s + \varepsilon_{z,j}$ and $u_j = s + \varepsilon_{u,j}$. Thus, the bankruptcy rate can be written as

Cross-sectional dispersion in returns. Let r_j denote the lenders' ex-post return from the contract signed with borrower j, conditional on that borrower j is still solvent (i.e., $j \in \mathcal{S}$), which is given by

$$r_j = \frac{\left[\alpha_j \rho e^{z_j} + (1-\alpha_j)e^{\hat{u}+\kappa z_j}\right] R_k}{1-n_j}. \tag{15}$$

Remember that $1-n_j$ is the amount of funds lent to borrower j, and the numerator is the repayment when the borrower is solvent (see Proposition 1). Then, the cross-sectional dispersion in ex-post returns, denoted by σ_p^2, is given as

$$\sigma_p^2 = \frac{\int_{j\in\mathcal{S}} (r_j - \bar{r})^2}{\int_{j\in\mathcal{S}} 1}. \tag{16}$$

where $\bar{r}$ is the average of r_j.

As can be seen from expression (xx), the cross-dispersion of r_j is due not only to the idiosyncratic shocks z_j but also to the heterogeneity in the borrowers' financial contracts (α_j, n_j). To single out the effects of the heterogeneity on dispersion of return, we consider the lenders' expected return from the contract signed with borrower j, which is given as

$$\hat{r}_j = \frac{\left\{\alpha_j \rho \mathbb{E}\left[e^{z_j} | j \in \mathcal{S}\right] + (1-\alpha_j)e^{\hat{u}} \mathbb{E}\left[e^{\kappa z_j} | j \in \mathcal{S}\right]\right\} R_k}{1-n_j}$$

Thus, the heterogeneity of expected return $\hat{r}_j$ is only due to the fact that borrowers made different financial decisions on n_j and α_j.

Therefore, the differential between realized return r_j and the market return $\bar{r}$ can be decomposed into two parts: $r_j - \hat{r}_j$ and $\hat{r}_j - \bar{r}$. The first part is due to the idiosyncratic shock z, while the second part reflects the heterogeneity in borrowers' financial decisions. The dispersion in ex-post returns can be decomposed as follows:[13]

$$\sigma_p^2 = \frac{1}{\int_{j\in\mathcal{S}} 1} \left(\underbrace{\int_{j\in\mathcal{S}} (r_j - \hat{r})^2}_{\text{idiosyncratic risk}} + \underbrace{\int_{j\in\mathcal{S}} (\hat{r}_j - \bar{r})^2}_{\text{heterogeneity}} \right). \tag{17}$$

$Prob(\varepsilon_{u,j} - \kappa \varepsilon_{z,j} < \hat{u} - (1-\kappa)s)$, where $\kappa \in (0,1)$. Thus, when the aggregate state s is low, more borrowers go bankrupt.

[13]Note that the expectation of the cross term $(r_j - \hat{r}_j)(\hat{r}_j - \bar{r})$ is zero.

Thus, to single out the effects of the heterogeneity in borrowers' financial decisions on dispersion in returns, we consider the dispersion in ex-ante returns defined as follows:

$$\sigma_h^2 = \int_0^1 (\hat{r}_j - \bar{r})^2 dj. \tag{18}$$

Market volatility. The time-series market volatility, denoted by σ_m^2, is the ex-ante variance of the market return. The market return of financial contract, denoted by $r_{m,s}$, is given as

$$r_m = \int_{j \in \mathcal{S}} \omega_j r_j, \tag{19}$$

where ω_j is the weight of contract signed with borrower j, defined as

$$\omega_j = \frac{1 - n_j}{\int_{j \in \mathcal{S}} (1 - n_j)}.$$

Thus, the market volatility is given as

$$\sigma_m^2 = var(r_m) \tag{20}$$

Financial condition. The economy-wide financial condition is captured by the spread between the aggregate return of investment, R_k, and the funding cost, R.

3.2 Numerical exercises

Parameter values. Following Bloom (2009), we assume that the aggregate and idiosyncratic risks have the same scales and set $\sigma_s = \sigma_\varepsilon = 0.1$, so that the steady state cross-sectional dispersion of ex-post returns is around 7%, which is inline with the observed stock return dispersion. The return of the storage technology, R, is set to one. The scalar A in the production function is set to one, and capital share θ is set to 0.25. Parameters ρ and μ (monitoring costs) are set respectively to 0.976 and 0.2, and thus the steady state default rate is around 2%, and the steady state external finance premium is 5%. The steady state wealth distribution is obtained by simulating the model without aggregate fluctuations. Parameter values are summarized in Table A1 in Appendix A1.

Correlations. Table 1 shows the correlations between the cross-sectional dispersion in ex-post returns (the time-series volatility of market return) and other measures. For each pair, the first columns show the coefficients estimated using actual data. To connect the model to empirical data, the same set of coefficients were also estimated using model-simulated data, shown in the second columns. As can be seen, the simulated values are in line with the data counterparts.

In the model, the correlation coefficient between market volatility and financial condition is almost one. This is due to the set up of the model. Note that in the model, financial condition is represented by external financing premium, i.e. $R_k - R$. The aggregate return R_k is endogenously determined by how the borrowers allocate their resources between ex-ante monitoring and production, that is, the average α. Market volatility also depends on the average α, as can be seen in Section 3.1. Thus, these two measures are almost perfectly correlated.

Table 1: Correlations

	I. Cross-sectional dispersion				II. Market volatility	
	Data (1)	Model (2)	Data (3)	Model (4)	Data (5)	Model (6)
Financial condition	0.770	0.7378			0.879	0.999
	(0.059)	(0.028)			(0.044)	(0.001)
Market volatility			0.728	0.7365		
			(0.063)	(0.0276)		
R-square	0.530	0.543	0.594	0.544	0.772	0.999
Observation	120	-	120	-	120	-

Notes: All variables are normalized to have a standard deviation of one. The actual dispersion and volatility are calculated using stock returns between 2003M01-2012M12. The basket of selected stocks is consistent with Bloom (2009). Dispersion in stock returns is the cross-sectional standard deviation of firm-level stock returns. The actual financial condition is the Chicago Fed National Financial Condition Index. The actual market volatility is the VXO index. The standard deviation is reported in the brackets below.

The effects of a decrease in aggregate productivity s. We assume that the economy is initially at the steady state. In the first period, the aggregate productivity drops to $s_1 = -2\sigma_s$, where σ_s is the standard deviation of s. After that, $s_t = 0$ for $t = 2, 3, 4....$

The left panel of Figure 2 plots the dynamics of the spread between aggregate capital return R_k and the borrowers' funding cost R. This spread is interpreted as ‘external finance premium’, which captures the aggregate financial condition. In the model, when borrowers’ financial conditions are worsened, to reduce the expected cost of bankruptcy (ex-post monitoring), borrowers relay more on ex-ante monitoring, which helps mitigate the information friction. However, since resources are spent to set up ex-ante monitoring, the borrowers’ investment decreases and then the aggregate capital return, R_k, increases.

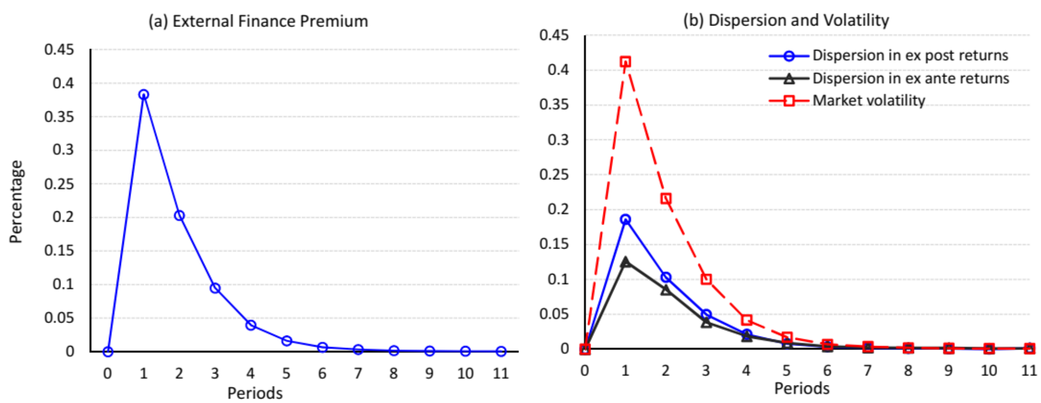

Figure 2: Response to an exogenous drop in the aggregate productivity

The lower panel of Figure 2 shows the dynamics of the cross-sectional dispersion in ex-post and ex-ante stock returns and market volatility. The detailed definitions of these measures can be found in the Section 3.1.

As shown in Figure 2, the cross-sectional dispersion in ex-post returns rises sharply in the second period and slowly comes back to its steady state after that. The increase in the dispersion in ex-post returns is due to two reasons: (i) when the borrowers’ financial conditions are worsened, the optimal contract calls for more intensive use of ex-ante monitoring and distributes more risks to the lenders (the first component of equation (17)); (ii) when the fraction of financially constrained borrowers increases, the heterogeneity in financial decisions increases, which leads to increase in the dispersion in ex-ante asset returns (the second component of equation (17)). As shown in the figure, the dispersion in ex-ante returns increases since the second period as expected, reflecting the increase in the heterogeneity in financial decisions.

Finally, note that all the responses are persistent even if the shocks do not have any persistence at all. The reason is that dispersion and market volatility are largely determined by the distribution of financial conditions of all the borrowers, and it takes time for financial conditions to recover after the negative shock.

4 Financial structure over the business cycle

An alternative explanation of our results is to consider the optimal contract as a mixture of standard equity and debt. Recall that a borrower can split their investment into parts and apply different monitoring technologies to each. Specifically, a fraction α of the borrower's investment is under ex-ante monitoring, and thus the return of which is observable to both the borrower and lender. The rest of the borrower's investment is not under ex-ante monitoring, and thus the return of which is only observable to the borrower.

Thus, a natural way to implement the optimal contract is to attribute all the observable revenues to the equity holders and a fixed amount of unobservable revenue to the debt holders as in the standard CSV models. This indicates that the cost of ex-ante monitoring is associated with issuing additional equity shares, while the cost of ex-post monitoring is associated with costly bankruptcy. Therefore, the weight of equity financing depends positively on the borrower's choice of monitoring scheme, α.

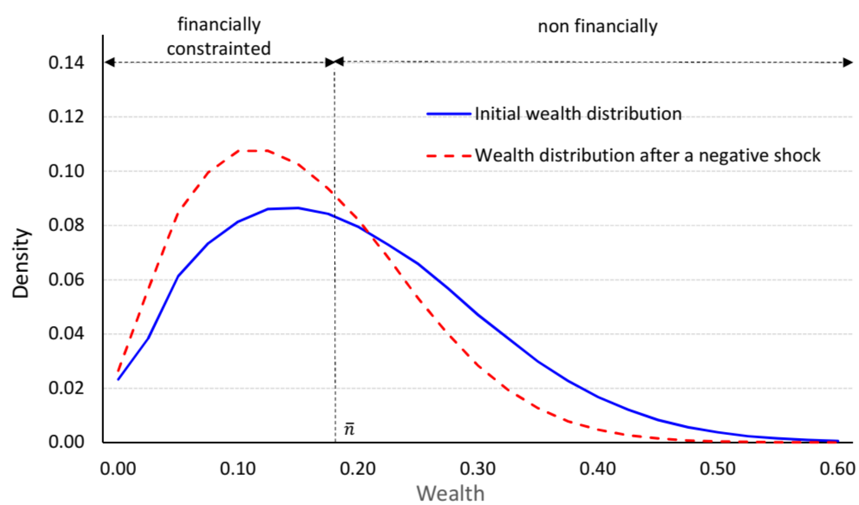

Figure 3: Declines in wealth and financial decisions

Following a negative shock, the borrowers' wealth distribution to the left as shown in Figure 3, and more borrowers become financially constrained. Thus, the optimal contracts call for more intensive uses of ex-ante monitoring, leading to a rise the average α. This implies that more equity financing is needed in the implementation of the optimal contracts. Our results are consistent with the empirical evidence provided by Covas and Den Haan (2011), Jermann and

Quadrini (2012) and Begenau and Salomao (2019).

5 Conclusion

We incorporate a CSV problem into a dynamic framework. The model deviates from classical CSV models by allowing for ex-ante monitoring, which refers to setting up a mechanism in advance that can credibly force the borrower to disclose the true investment outcome to the lender. Ex-ante monitoring is costly, but can mitigate the information friction.

When the borrower's financial condition is worsened after a negative shock, the expected cost of bankruptcy (ex-post monitoring) increases. The optimal contract that minimizes monitoring costs calls for more intensive use of ex-ante monitoring, requiring the repayment be contingent on the information revealed. Thus, during economic downturns the optimal contract becomes more equity-like.

The financial contract can be interpreted as (outside) equity. In this case, as the optimal contract becomes more equity-like during downturns, the lender takes more aggregate and idiosyncratic risks. Thus, the cross-sectional dispersion and market volatility in stock returns rise. Our model predicts countercyclical movements in financial conditions, and cross-sectional dispersion and market volatility in stock returns. Calibrated at a steady state, the model successfully generates correlation coefficients between these measures that align quantitatively with empirical data.

An alternative interpretation is to consider the contract as a mixture of standard debt and equity. In this case, as the optimal contract becomes more equity-like during downturns, the implementation of the contract calls for more intensive use of equity financing.

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Appendices

A1 Empirical analysis

Table A1: Parameter values

	Value	Description
σ_s	0.1	Aggregate uncertainty
σ_ε	0.1	Idiosyncratic uncertainty
R	1.000	Return of return of the storage technology
ρ	0.976	$1-\rho$ is the ex-ante monitoring cost
μ	0.2	Ex-post monitoring cost
θ	0.25	Capital share in the production function

A2 Mathematical derivations and proofs

A2.1 Proof of Lemma 1

For any z, such that $x(z) \geq \alpha\rho e^z R_k$, there is a critical value of u, i.e. $\hat{u}(z)$.

First, note that if $u < \hat{u}(z)$ the repayment $x(z)$ is not feasible. Then, $(-\infty, \hat{u}(z)) \subseteq D(z)$. Suppose that there is $u'(z) \in D(z)$ and $u'(z) > \hat{u}(z)$. Note that the repayment $x(z)$ is feasible at $u = u'(z)$. Thus, it is optimal to reduce $x(z)$ and take $u'(z)$ out of monitoring region, while keeping the expected repayment to lender unchanged. Thus, $D(z) \subseteq (-\infty, \hat{u}(z))$. In sum, $D(z) = (-\infty, \hat{u}(z))$.

Second, if for any $u \in D(z)$, $r(z,u) < [\alpha\rho e^z + (1-\alpha)e^u]R_k$, it is optimal to increase $r(z,u)$ to $[\alpha\rho e^z + (1-\alpha)e^u]R_k$ and reduce $x(z)$ and also the monitoring region $D(z)$, while keeping the expected repayment to lender unchanged. Thus, $r(z,u) = [\alpha\rho e^z + (1-\alpha)e^u]R_k$ must hold.

Third, now assume that for some z_1, $x(z_1) < \alpha\rho e^{z_1} R_k$. It is always optimal to increase $x(z_1)$ to $\alpha\rho e^{z_1} R_k$ and reduce $x(z)$ and also the monitoring region $D(z)$ for the other states of z, while keeping the expected repayment to lender unchanged. Thus, $x(z) \geq \alpha\rho e^z R_k$ for any z.