Demand Heterogeneity, Firm Reallocation, and the Labor Share

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Abstract

Preferential access to capital can shift production across firms and change the aggregate labor share. Whether it rises or falls depends on which competitors lose revenue shares as selected firms expand. We develop a method for assessing this effect with unobserved demand groups and apply it to Chinese manufacturing data. Under the grouping constructed from firms' cost ratios, the model predicts a higher labor share in every year following a standardized reduction in selected firms' relative capital costs. In later years, alternative groupings allowed by the cost-ratio information can reverse the prediction. Holding selected firms' shares of group value added at their fitted levels restores a positive effect under the same demand responses. Our paper shows how demand substitution and the composition of expanding firms jointly shape the distributional consequences of capital reallocation.

Keywords: Labor share; misallocation; demand heterogeneity; substitution patterns; productivity.

JEL codes: C38, D24, D33, O47.

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1 Introduction

Policies and frictions that change firms' relative costs of capital shift economic activity across firms. Research on misallocation has studied the implications of this reallocation for aggregate productivity (Hsieh and Klenow, 2009; Baqaee and Farhi, 2020); work on the labor share shows that shifting value added among firms with different wage shares can also change the aggregate share paid to workers (Autor et al., 2020; Smagghue, 2022). For a targeted reduction in the cost of capital, the direction of the wage-share effect depends on which firms lose revenue shares as recipients expand and on how strongly buyers substitute among their products. Enterprise surveys record firm inputs and value added, but seldom identify these groups of close competitors. We study the wage-share consequences of capital reallocation when the relevant demand groups are unobserved.

Chinese manufacturing provides a useful application of this general problem. State-owned firms have had better access to bank credit than private firms (Song et al., 2011), and loans and subsidies have been instruments of industrial policy (Aghion et al., 2015). The industrial-enterprise surveys report wages, capital, and value added for a large set of firms over multiple years. These features let us examine how a standardized change in selected firms' relative capital costs shifts the aggregate wage share, measured as total reported wage payments divided by total value added.

Changes in the aggregate wage share combine changes within firms with shifts in their shares of value added. Financial conditions can affect both channels: David et al. (2024), for example, show how international risk sharing raises wage shares within firms while reallocating activity toward firms with lower wage shares. We focus on the reallocation channel generated by a change in relative capital costs. We represent substitution patterns as demand groups in a nested-CES economy. A recipient gains revenue share relative to firms in the same group, and the strength of this response varies across groups. The aggregate effect depends on recipients' wage shares relative to those of their competitors and on how strongly each group responds. Our method evaluates that effect while allowing demand-group assignments to remain uncertain.

We analyze a standardized counterfactual that reduces the capital-input wedge for a prespecified set of firms. The proportional wedge change alters firms' effective capital costs; the common rental rate adjusts in equilibrium. The observed enterprise records determine the baseline allocation, while an industry-relative capital-intensity rule selects recipients and a five-percent wedge reduction specifies the intervention. These choices give the same transparent experiment in every annual sample. Within each demand group, recipients gain revenue relative to nonrecipients. A common wedge change for all firms in one group leaves that group's relative revenues unchanged. Firm-level wage shares remain fixed in the experiment, so the aggregate response comes from differences in wage shares within groups, weighted by the resulting revenue shifts.

Our methodological result identifies two pieces of group-level information that make this finite counterfactual tractable: recipients' initial share of each group's value added and the group's revenue-response coefficient. Together with the fitted group masses within cost-ratio bins, this information makes the wage-share change linear in the unknown assignments. Individual firm assignments and group value-added totals can remain uncertain. Linear programs give the smallest and largest effects over the resulting fractional-assignment set, and monotonicity extends the calculation to intervals for the response coefficients. The method thus connects information about recipients' economic weight among their competitors to the direction and magnitude of the aggregate wage-share effect.

We apply the method to all retained observations in ten annual samples of Chinese industrial enterprises from 1998 to 2007, comprising 1,711,329 firm-years after common cleaning. Mixtures fitted to log sales-to-cost ratios, combined with input expenditure ratios and economic restrictions, provide a benchmark grouping and group response coefficients. We then vary assignments and parameters within stated restrictions, keeping each year's observed firms and counterfactual recipient rule fixed. A recipient has a capital-to-value-added ratio above the unweighted median of its four-digit industry. The five-percent change and this threshold define the counterfactual, allowing comparisons across years on a common basis.

The 2007 benchmark implies an increase of 0.362 percentage points in the aggregate wage share. With only the fitted group masses within cost-ratio bins held fixed, admissible assignments produce both positive and negative responses, even when they preserve each industry's benchmark effect of complete wedge removal. Holding the mixture-implied benchmark recipient share of baseline value added fixed in every group gives the sharp conditional range of 0.090 to 0.633 points. Its lower endpoint is positive in every annual sample. The comparison shows how information about the composition of recipients within groups resolves an ambiguity left by the complete-removal benchmark.

The group revenue responses also matter. At fixed benchmark recipient shares, smaller deviations in the mapping from observed costs and inputs to response coefficients retain a positive lower bound in every year. Wider deviations permit a negative response under a common labor output elasticity within each industry. Allowing recipient shares and parameters to vary together leaves the direction unresolved at the reported bounds. These results locate the information required to infer a sign: evidence about who competes with the recipients must be paired with sufficiently precise restrictions on how those groups respond.

We examine the role of measurement and recipient composition among 51,959 firms observed in both the industrial and tax surveys in 2007. For the capital-to-value-added recipient rule, the industrial and tax cost reports give benchmark increases of 0.397 and 0.540 percentage points, and both yield positive conditional ranges at their respective benchmark recipient shares. Selecting firms by capital per worker instead makes both benchmark effects negative; the conditional ranges for that rule include zero. The rules select the same number of firms but different amounts of value added

and firms with different wage shares. Fixing both rules' benchmark recipient shares ranks their effects within each reporting source, even though the conditional range for the capital-per-worker rule crosses zero.

Related literature. The paper connects the misallocation and labor-share literatures through a question about missing demand information. Hsieh and Klenow (2009) quantify factor distortions using Chinese and Indian manufacturing data, and Baqaee and Farhi (2020) derive general-equilibrium aggregation formulas under heterogeneous distortions and substitution patterns. Their framework takes the relevant substitution structure as a primitive; our finite wage-share calculation examines uncertainty about which firms are close substitutes. Smagghue (2022) shows how heterogeneous distortions change the aggregate labor share through reallocation, while David et al. (2024) connect international diversification to opposing within-firm and reallocation effects. Using later Chinese tax-survey data, Chen et al. (2026) study preferential credit across sectors with heterogeneous markups and evaluate productivity and welfare. We characterize the group-composition and revenue-response information needed to determine the wage-share effect of a specified finite change in capital wedges.

David and Venkateswaran (2019) distinguish sources of capital misallocation using investment and value-added dynamics. Their joint use of moments separates adjustment costs, imperfect information, and other firm-specific factors that can have similar implications for the dispersion in value added relative to capital. We examine a complementary information problem. Even with the baseline allocation and the capital-wedge change specified, uncertainty about the recipients' competitors can permit opposing wage-share responses. Recipients' shares of group value added provide the composition restriction that makes our finite response linear; group revenue-response coefficients determine how strongly that composition affects the outcome.

Work on latent groups provides a further point of comparison. Asturias and Rossbach (2023) recover groups in factor cost shares and bound how such heterogeneity changes estimates of the gains from eliminating misallocation. Almagro et al. (2026) estimate demand nests from product choices, characteristics, and prices across markets. We use cost-ratio mixtures and an explicit economic mapping to construct a benchmark grouping in enterprise data, and then examine how the prediction changes across assignments preserving the fitted group masses within cost-ratio bins. The restrictions on recipients' group value-added shares connect this grouping exercise to the wage-share effect. They also make explicit which information about close substitutes would sharpen the result.

The technical contribution relates to methods that study counterfactual conclusions under incomplete structural information. Adão et al. (2017) characterize trade counterfactuals through reduced factor demand; Andrews et al. (2020) connect descriptive statistics to the sensitivity of structural estimates; and Christensen and Connault (2023) develop bounds for counterfactuals with latent heterogeneity. Our setting yields an exact reduction for a finite, distributional counterfactual: fixing

recipients' baseline value-added shares in demand groups removes the assignment-dependent normalization, and coefficient monotonicity converts parameter intervals to endpoint linear programs. Luo et al. (2026) similarly study incomplete information about production-network links. The link between the missing group assignment and the observed wage-share contrast gives the resulting bounds their economic interpretation.

Finally, the application addresses the mapping from reported costs to economic parameters. Finite-mixture methods discipline the statistical grouping (Bonhomme et al., 2022; Bonhomme and Weidner, 2022), while research on misallocation and markups stresses that measurement and production assumptions affect structural interpretations of enterprise records (Bils et al., 2021; Ruzic and Ho, 2023; De Ridder et al., 2026). The matched-survey analysis uses two contemporaneous cost reports for the same firms while holding baseline value added, wages, capital, and the recipient set fixed. It shows how changes in the cost signal affect the benchmark grouping and the implied wage-share response.

2 Demand Heterogeneity and Capital Reallocation

2.1 Demand Structure, Production, and Capital Wedges

Each firm supplies a variety in one of several CES nests within its industry. A competitive final-good producer combines the nest composites with a Cobb–Douglas aggregator and varieties within each nest with CES:

$$Y = \prod_g Y_g^{\beta_g}, \qquad Y_g = \left(\sum_{i \in g} q_i^{(\epsilon_g - 1)/\epsilon_g} \right)^{\epsilon_g/(\epsilon_g - 1)}, \qquad \beta_g > 0, \quad \sum_g \beta_g = 1, \quad \epsilon_g > 1. \tag{1}$$

Here Y is final output, Y_g is the quantity composite for nest g, and q_i is firm i's output. The weight β_g is the nest's share of final expenditure, and ϵ_g is the within-nest elasticity of substitution. We use *group* and *nest* interchangeably below. Firms take group expenditure and its price index as given. Their value-added technologies are $q_i = A_i K_i^{1-\alpha_s} L_i^{\alpha_s}$, with A_i denoting productivity, K_i capital, and L_i effective labor. The labor output elasticity $0 < \alpha_s < 1$ is common within industry s. Demand weights within groups are absorbed into productivity. Effective labor and capital are freely mobile, have common prices w, r, and have fixed aggregate supplies $\bar{L}, \bar{K}$. For each candidate assignment, baseline group expenditure shares are calibrated from the value added allocated to that group and held fixed during its counterfactual. These assumptions extend the allocation framework of Hsieh and Klenow (2009) to heterogeneous CES nests.

Write p_i for the firm's output price, $V_i = p_i q_i$ for nominal value added, and $W_i = w L_i$ for labor payments. A firm retains $H_i V_i$ after its revenue wedge and pays $\kappa_i r$ per unit of capital services, where

$\kappa_i > 0$ is its capital-input wedge:

$$\Pi_i = H_i V_i - W_i - \kappa_i r K_i, \qquad H_i > 0. \tag{2}$$

The term $\kappa_i r$ is the firm's marginal effective price of capital services. More favorable financing terms can enter through a lower κ_i when they reduce this price. A revenue subsidy has $H_i > 1$; a capital subsidy has $\kappa_i < 1$. The counterfactual below specifies a change in this price wedge and its transfer counterpart. Government net receipts $\mathcal{T} = \sum_i [(1 - H_i)V_i + (\kappa_i - 1)rK_i]$ are transferred to the household, with negative receipts financed by a lump-sum tax. The household owns firms and factors, so its expenditure satisfies

$$PY = w\bar{L} + r\bar{K} + \sum_i \Pi_i + \mathcal{T} = \sum_i V_i. \tag{3}$$

Here P is the final-good price index. Wedge payments cancel between firms and the household budget, so total final expenditure equals nominal value added. This transfer rule treats wedge receipts as redistribution within the economy. Wedges representing real resource losses would instead require a different resource constraint (Baqaee and Farhi, 2020).

An equilibrium consists of prices and allocations satisfying firm profit maximization, competitive final-good production, the household budget, and factor-market clearing, $\sum_i L_i = \bar{L}$ and $\sum_i K_i = \bar{K}$. The final good is consumed by the household.

Define the CES markup parameter $\mu_g = \epsilon_g/(\epsilon_g - 1) > 1$. Profit maximization implies

$$\ell_i \equiv \frac{W_i}{V_i} = \frac{\alpha_s H_i}{\mu_g}, \qquad \frac{rK_i}{V_i} = \frac{(1 - \alpha_s)H_i}{\mu_g \kappa_i}. \tag{4}$$

The first ratio is the firm-level wage share ℓ_i, its wage bill as a share of value added. The second measures the capital payment at the common rental rate relative to value added. Holding technology, demand, and the revenue wedge fixed, a higher capital wedge lowers this ratio. Because α_s and μ_g are common within a nest, the calibrated revenue wedge H_i accounts for variation in firm wage shares within that nest. The intervention holds H_i fixed, so these wage-share differences remain as revenue is reallocated. The markup μ_g compares the price received by the firm, $H_i p_i$, with private marginal cost; the ratio of the observed price p_i to that cost is μ_g/H_i. The measured factors are inputs to the maintained value-added technology. Its interpretation and the mapping from accounting measures to benchmark parameters are specified in the data section.

We analyze a proportional reduction in the capital wedge for a prespecified set of firms:

$$\kappa_i(\tau) = \kappa_i \exp(-\tau e_i), \qquad H_i(\tau) = H_i, \qquad e_i \in \{0, 1\}, \quad \tau > 0. \tag{5}$$

The fixed indicator e_i equals one for a recipient and zero otherwise. A relief fraction r_c corresponds to $\tau = -\log(1 - r_c)$, so that a recipient's wedge is multiplied by $1 - r_c$. The common rental rate r adjusts to clear the capital market. The resulting change in a recipient's effective capital price therefore also reflects the equilibrium change in r. Firms, technology, demand parameters, and aggregate factor supplies remain fixed, and transfers are recomputed at the new allocation. We compare two static equilibria with the same active firms and aggregate factor supplies, allowing capital services and labor to move across firms. Modeling investment dynamics additionally requires capital adjustment and expectations, as in David and Venkateswaran (2019). The application uses the equilibrium comparison to determine the information needed for a wage-share response.

2.2 Reallocation and the Wage Share

To distinguish the channels through which an aggregate share changes, let $s_i^j = V_i^j / \sum_h V_h^j$ be firm i's share of total value added and $\ell_i^j = W_i^j / V_i^j$ its wage share in allocation $j \in \{0, 1\}$. For a fixed set of firms, the aggregate change satisfies

$$LS^1 - LS^0 = \underbrace{\sum_i (s_i^1 - s_i^0)\ell_i^0}_{\text{reallocation}} + \underbrace{\sum_i s_i^1(\ell_i^1 - \ell_i^0)}_{\text{within-firm changes}}. \tag{6}$$

The first term measures shifts in firms' value-added weights, evaluated at their initial wage shares. The second measures changes in firm wage shares, using the new value-added weights. This exact accounting identity organizes the two channels emphasized in the labor-share literature (Autor et al., 2020; David et al., 2024). Our intervention holds H_i, α_s, and μ_g fixed, so (4) implies $\ell_i^1 = \ell_i^0$: the modeled effect operates entirely through reallocation. Since the group expenditure shares β_g are fixed, this reallocation takes place within demand groups.

Let $\omega_i = V_i / \sum_{h \in g} V_h$ denote baseline within-group revenue weights. Common demand and factor-price terms cancel from relative revenues, giving

$$\omega_i(\tau) = \frac{\omega_i \exp(\tau \lambda_g e_i)}{\sum_{h \in g} \omega_h \exp(\tau \lambda_g e_h)}, \qquad \lambda_g = \frac{1 - \alpha_s}{\mu_g - 1}. \tag{7}$$

The coefficient λ_g governs the relative-revenue response to a proportional reduction in the capital wedge. For a recipient and a nonrecipient in the same nest, their revenue ratio is multiplied by $\exp(\tau\lambda_g)$. The response is stronger when capital has a larger output elasticity or varieties within the nest are more substitutable. The first-order condition in (4) keeps each firm's wage share ℓ_i unchanged. Labor's share therefore changes through the reallocation of revenue within groups:

$$LS(\tau) = \sum_g \beta_g \frac{E_g[\ell_i \exp(\tau \lambda_g e_i)]}{E_g[\exp(\tau \lambda_g e_i)]}, \qquad LS'(0) = \sum_g \beta_g \lambda_g \operatorname{Cov}_g(\ell_i, e_i). \tag{8}$$

Here LS is aggregate wage payments divided by aggregate value added. Expectations and covariances use baseline revenue weights within a group. The first expression evaluates the finite intervention; the derivative describes its local response at the baseline. The covariance is positive when recipients have a higher average wage share than nonrecipients within their nest. Appendix A derives the exact finite formula and its extension to proportional interventions on several inputs.

For a binary recipient indicator, the mechanism has a particularly direct interpretation. Let $p_g = E_g[e_i]$ be recipients' share of group value added. Their counterfactual revenue share is

$$p_g(\tau) = \frac{p_g \exp(\tau\lambda_g)}{1 - p_g + p_g \exp(\tau\lambda_g)}. \tag{9}$$

This share rises when a nest contains both recipients and nonrecipients; its change measures revenue reallocation toward recipients. For a group containing both recipient types, define $d_g = E_g[\ell_i \mid e_i = 1] - E_g[\ell_i \mid e_i = 0]$. The exact aggregate wage-share change is

$$LS(\tau) - LS(0) = \sum_g \beta_g d_g [p_g(\tau) - p_g]. \tag{10}$$

Each term combines the group's expenditure weight β_g, the difference in average wage shares between recipients and nonrecipients d_g, and the change in recipients' revenue share. Groups with $p_g = 0$ or 1 contribute zero. A reduction in the capital wedge raises recipients' share of revenue in nests containing both types. The group wage share rises when recipients have a higher average wage share than nonrecipients in the same nest, and falls when their average wage share is lower. The coefficient λ_g determines how far revenue shifts toward recipients. Nest assignments determine which firms enter each wage-share comparison and how much revenue is reallocated within each group. For example, recipients can have a higher average wage share than their industry as a whole yet a lower average than nonrecipients in their demand group. Reallocation toward them then lowers that group's wage share. The relevant comparison is the one between firms whose revenue weights change against each other.

The outcome is wages as a share of nominal value added. With fixed aggregate effective labor, the real wage satisfies $w/P = LS\,Y/\bar{L}$; workers' purchasing power therefore also depends on the response of final output Y.

2.3 Complete Wedge Removal and Partial Changes in Capital Wedges

A useful comparison sets $H_i' = \kappa_i' = 1$ for all firms while retaining monopolistic competition. The final firm-level wage share is then common within each group, so the aggregate wage share after removing

both wedges is

$$LS^{\text{rem}} = \sum_g \beta_g \frac{\alpha_{s(g)}}{\mu_g}. \tag{11}$$

Here $s(g)$ denotes the industry containing group g. Removing the revenue and capital wedges leaves the firm-level wage share $\alpha_{s(g)}/\mu_g$ within each group, with the group's expenditure weight unchanged. This removal benchmark summarizes technology, demand parameters, and group expenditure shares. The finite experiment additionally depends on the association between recipient status and wage shares within groups, as (10) shows. Candidate assignments can consequently agree on the removal benchmark and imply different finite responses. The application compares these information states on the same firms and recipient rule. The remaining removal accounting identities are collected in Appendix A.

3 Wage-Share Bounds with Unknown Nest Assignments

3.1 Admissible Nest Assignments

The observed records contain value added $V_i > 0$, wage payments $W_i > 0$, capital $K_i > 0$, and a fixed recipient indicator e_i. Let a_{ig} be the fraction of the mass represented by record i assigned to nest g. A record can be assigned only to nests in its industry. Within each bin b of the log sales-to-cost ratio, impose

$$a_{ig} \geq 0, \qquad \sum_g a_{ig} = 1, \qquad \sum_{i \in b} a_{ig} = m_{bg}. \tag{12}$$

The first two restrictions allocate each record in full; the last fixes the mass assigned from bin b to group g. These margins m_{bg} are measured in firm-count units and sum to the number of records in each bin; each group has positive total mass. In the application, the first-stage distribution fit and maintained economic mapping supply benchmark parameters and posterior component probabilities. Sums of these probabilities determine the masses. Write $\mathcal{A}$ for this transportation polytope, augmented when stated by additional linear economic restrictions. We retain the bin-level group masses and the stated restrictions on exposure and parameters while allowing firms with different value added, wage shares, and recipient status to change nests within each bin. These restrictions preserve bin-level group masses; an exact within-component distribution would require further restrictions, as discussed in Appendix C.

Fractional assignments treat each record as a mass of observationally identical firms that can be distributed across candidate nests. Each assignment preserves the record's total weight and the group masses within cost-ratio bins. Its group value added M_g, group wage bill L_g, recipient value added

U_g, and recipient wage bill C_g are

$$M_g = \sum_i a_{ig} V_i, \quad L_g = \sum_i a_{ig} W_i, \quad U_g = \sum_i a_{ig} V_i e_i, \quad C_g = \sum_i a_{ig} W_i e_i. \tag{13}$$

Its baseline expenditure share is $\beta_g(a) = M_g(a)/T$, where $T = \sum_i V_i$ is total observed value added. The group wage bill L_g aggregates payments; the production input L_i in Section 2 measures labor quantities. For each candidate assignment and parameter vector, we calibrate revenue and capital wedges and productivity to reproduce the observed baseline value added, wages, and capital, using the construction in Appendix A. Revenue wedges, productivity, and the calibrated group expenditure shares remain fixed during that economy's intervention; capital wedges change according to (5). Group expenditure shares can differ across admissible economies, even though they are fixed during each economy's counterfactual. We also compare the fractional results with assignments that keep each firm intact.

For a specified shock and coefficient vector, write $q_{ig} = \exp(\tau \lambda_g e_i)$, $Q_g^V = \sum_i a_{ig} V_i q_{ig}$, and $Q_g^W = \sum_i a_{ig} W_i q_{ig}$. The multiplier q_{ig} tilts each record's baseline revenue weight. The ratio Q_g^W / Q_g^V is the resulting group wage share, so the aggregate counterfactual wage share is

$$F_\tau(a) = \frac{1}{T} \sum_g \frac{M_g(a) Q_g^W(a)}{Q_g^V(a)}, \qquad F_0 = \frac{\sum_i W_i}{T}. \tag{14}$$

The group share M_g/T weights the counterfactual group wage share. The tilted sums use observed baseline value added and wage payments. Every assignment has the same baseline aggregate share F_0, so optimizing F_τ or its change yields the same endpoint assignments.

3.2 Recipient Shares and Sharp Bounds

Suppose the value-added share of recipients in each group is supplied as $p_g \in [0,1]$. Define

$$\mathcal{A}(p) = \{a \in \mathcal{A} : U_g - p_g M_g = 0 \text{ for every } g\}, \qquad \mathcal{R}_g(a) = C_g(a) - p_g L_g(a).$$

The share p_g weights firms by value added, whereas the original margins m_{bg} weight them by count. The equality $U_g = p_g M_g$ fixes recipients' economic weight in a group. The cross-moment $\mathcal{R}_g$ measures recipients' wage payments in excess of their value-added share of the group wage bill. The set $\mathcal{A}(p)$ collects all assignments meeting these restrictions.

Proposition 1 (Exact bounds with group exposure information)**.** *Let exposure be binary and $\mathcal{A}(p)$ be nonempty. For fixed finite response coefficients and shock τ, define*

$$\gamma_g = \exp(\tau \lambda_g) - 1, \qquad k_g(\lambda_g, p_g) = \frac{\gamma_g}{1 + \gamma_g p_g}.$$

The exact finite wage-share change is linear in the assignment variables:

$$\Delta F(a;\lambda,p) = F_\tau(a) - F_0 = \frac{1}{T}\sum_g k_g(\lambda_g, p_g)\mathcal{R}_g(a). \tag{15}$$

Its sharp lower and upper bounds over the fractional-assignment set $\mathcal{A}(p)$ are attained by linear programs.

Proof. Binary exposure implies $Q_g^V = M_g + \gamma_g U_g = M_g(1+\gamma_g p_g)$ and $Q_g^W = L_g + \gamma_g C_g$. Substitution in (14) cancels M_g; subtracting the common baseline gives (15). The denominator is positive, and the objective is linear on a compact feasible polytope. □

Fixing the recipient share p_g in each nest removes the assignment-dependent denominator from the exact finite response. This condition makes the response linear in the remaining assignment variables. The term γ_g is the excess multiplier on recipients' relative revenue, and k_g adjusts it for their initial value-added share. The LP endpoints are the smallest and largest responses consistent with the stated information and the fractional-assignment formulation.

Individual labels and group expenditure totals need not be supplied separately. In particular configurations, the exposure restrictions themselves determine group totals, as in the two-group example in Appendix A. The resulting association $\mathcal{R}_g$ equals $M_g p_g(1-p_g)d_g$ when both exposure types are present, connecting the LP directly to the mechanism in (10). Its sign depends on observed wage shares and their admissible assignment to nests. Appendix A gives a small example with fixed exposure shares and opposite responses.

Fixing M_g and U_g is a stronger sufficient condition for linearity; additionally fixing L_g is unnecessary. At unchanged parameters, fixing M_g also fixes the removal benchmark. Fixed p_g and fixed M_g are generally nonnested restrictions. For nonbinary exposure, a known M_g/Q_g^V similarly yields a linear target, and category-specific exposure shares provide this ratio for any stated shock.

3.3 Joint Bounds under Parameter Uncertainty

Let $\Lambda = \prod_g [\lambda_g^-, \lambda_g^+]$ be a compact Cartesian product of coefficient intervals, with $0 \le \lambda_g^- \le \lambda_g^+ < \infty$. The assignment restrictions are held fixed as coefficients vary.

Proposition 2 (Joint bounds over response-coefficient ranges)**.** *Let G denote the number of active groups. For $\tau > 0$, the sharp lower endpoint over $\mathcal{A}(p) \times \Lambda$ is*

$$\min_{a\in\mathcal{A}(p),\,\lambda\in\Lambda} \Delta F = \min_{v\in\{-,+\}^G} \min_{a\in\mathcal{A}(p)} \frac{1}{T}\sum_g k_g(\lambda_g^{v_g}, p_g)\mathcal{R}_g(a). \tag{16}$$

The upper endpoint has the analogous maximum representation. Each inner problem is a linear program. When restrictions separate by industry, vertices and endpoints can be computed industry by industry.

Proof. For fixed $p \in [0,1]$,

$$\frac{\partial k_g(\lambda, p)}{\partial \lambda} = \frac{\tau \exp(\tau\lambda)}{[1 + p\{\exp(\tau\lambda) - 1\}]^2} > 0.$$

For each assignment, the minimizing coefficient is the lower endpoint if $\mathcal{R}_g \geq 0$ and the upper endpoint if $\mathcal{R}_g < 0$; maximization reverses this choice. Minimizing over the finite vertex set and compact assignment set in either order proves the result. □

The vector v chooses the lower or upper coefficient endpoint for every group. Because k_g increases with λ_g, a larger response coefficient raises positive contributions and makes negative contributions more negative. The result reduces a continuous coefficient problem to endpoint LPs. The application has at most six groups per industry, permitting enumeration within each industry. In larger group systems the number of vertices can grow exponentially; sign restrictions on $\mathcal{R}_g$ can reduce the required combinations.

Economic parameter regions also encode restrictions shared across groups. Here $\lambda_g = (1 - \alpha_s)/(\mu_g - 1)$ uses one technology parameter α_s per industry. Taking the marginal range of each λ_g produces a rectangular region containing all coefficient vectors allowed by the common-technology restriction. The proposition gives sharp bounds for this rectangle; it can be wider than the economically admissible parameter set. A positive lower bound over the rectangle therefore establishes a positive response under the common-technology restriction. An outcome is feasible under common industry technology when its assignment satisfies the stated moment restrictions and its response coefficients come from admissible nest markups and one labor output elasticity α_s shared within each industry. To establish a negative response, we evaluate assignments and parameters satisfying these restrictions jointly. Appendix A describes the mapping and the uniform centered bound that can be used before vertex enumeration.

3.4 Bounds with Incomplete Exposure Information

Exposure bands specify a lower share p_g^- and an upper share p_g^+ for each group. The inequalities $p_g^- \leq U_g/M_g \leq p_g^+$ give linear restrictions $p_g^- M_g \leq U_g \leq p_g^+ M_g$, while leaving the exact objective (14) nonlinear. The same coefficient-endpoint reduction remains valid because, at a fixed assignment, the sign of the derivative of the group response with respect to λ_g is the sign of $C_g M_g - L_g U_g$. This expression measures the same association between recipient status and firm-level wage share, now using the exposure share implied by that assignment. At each coefficient vertex, perspective McCormick relaxations of the finite target and centered coefficient envelopes provide valid outer bounds. Exact evaluations at feasible assignments provide attained values. Appendix B gives the relaxations, their error bounds, and the domain subdivisions that preserve coverage.

Write L_*, U_* for the sharp endpoints, L_o, U_o for valid outer bounds, and L_i, U_i for attained lower and upper values. Then

$$L_o \leq L_* \leq L_i, \qquad U_i \leq U_* \leq U_o.$$

Outer bounds enclose every admissible response; attained values are responses delivered by particular admissible assignments. Thus these inequalities bracket each unknown sharp endpoint from both sides. The computational gaps are $L_i - L_o$ and $U_o - U_i$; they measure accuracy in locating the endpoints, separately from the width of the admissible economic range. A positive outer lower bound establishes a positive response throughout the set. A negative feasible response establishes that a negative outcome is attainable. A negative outer lower bound paired with a positive attained lower value leaves the sign unresolved.

In the application, we combine baseline firm value added and wage payments with a fixed recipient rule and wedge change, group masses within cost-ratio bins, and restrictions on group exposure and responses. With fixed exposure shares, the assignment problems at the coefficient vertices yield industry endpoints that aggregate to the economy-wide range. With exposure bands or unrestricted exposure shares, we report valid outer bounds alongside the responses attained by feasible assignments. The results distinguish these economic ranges from the remaining optimization gaps and identify outcomes that satisfy the common industry technology restriction.

4 Data and Benchmark Parameter Estimates

4.1 Manufacturing Data and Annual Samples

We use the Chinese industrial-enterprise database for 1998–2007. The available extract contains comparable gross output, intermediate expenditure, wages, sales, and corresponding business costs over this period. Earlier records lack the required common fields, and comparable intermediate expenditure is unavailable after 2007. We use the full retained sample in each year, without requiring firms to appear in adjacent years, and repeat the counterfactual analysis in ten annual cross sections.[2]

For firm i, gross value added is $V_i = Q_i - M_i$, where Q_i is current-price gross output and M_i is intermediate expenditure. We measure the aggregate wage share as $\sum_i W_i / \sum_i V_i$, using accrued wage payments W_i. Reported industrial value added, VAT, and accrued employee welfare benefits are retained as separate fields. Capital K_i is the supplied constructed stock. We normalize the benchmark rental rate to $r_0 = 0.2$ per supplied capital unit. Appendix F examines alternative capital valuations and distinguishes this normalization from an estimated user cost.

[2] David and Venkateswaran (2019) use the same survey family to study capital misallocation over 1998–2009. Their linked panel uses book capital and value added constructed as a constant fraction of sales. Our wage-share calculation uses reported output less intermediate expenditure, the supplied constructed capital stock, and annual samples requiring comparable wage and paired sales–cost fields. The variable requirements and estimation samples consequently differ.

Eligibility requires an unambiguous legal identifier and normalized name, positive core production and input variables, positive value added, positive paired sales and costs, and a uniquely harmonized manufacturing industry. Writing S_i for the paired reported sales and C_i for their corresponding business costs, sales and production must satisfy $0.5 \leq Q_i/S_i \leq 2$. The industry concordance is applied before estimation and record linkage. Sales and costs use product-sales fields in 1998–2000 and 2002–2003, and principal-business fields in 2001 and 2004–2007. Each ratio uses its corresponding reported cost field.

One tail rule applies to every annual sample. Within year and harmonized two-digit industry, we calculate the first and 99th percentiles of $\log(W/Q)$, $\log(r_0K/Q)$, $\log(V/Q)$, and $\log(S/C)$. We remove a record if any ratio lies strictly outside its interval. Cutoffs are computed once on the eligible sample. The rule addresses extreme input and cost intensities while retaining large firms with ordinary ratios. Appendix C gives the variable definitions, coverage chain, and benchmark-estimation procedure.

The annual samples contain 112,707 to 274,443 firms and together comprise 1,711,329 firm-year observations. They retain about 93 percent of eligible firms and 91–94 percent of eligible value added; Table C.1 reports the annual coverage chain. Every retained observation enters estimation and aggregation. We present detailed results for 2007, the last year with comparable production fields and the year available for the matched tax-survey comparison; this sample contains 425 harmonized four-digit industries. We report the same counterfactual for all ten years. A parallel winsorized series retains every eligible observation and clips the same ratios at the same cutoffs; Appendix F uses it to assess the tail sensitivity of the benchmark mapping and removal benchmark.

4.2 Cost Ratios, Input Parameters, and Benchmark Demand Groups

We fit the distribution of the log sales-to-cost ratio $X_i = \log(S_i/C_i)$ within each industry-year. Interpreting components of this distribution as demand nests requires a restriction connecting reported costs to private variable cost in the value-added model:

$$C_i^v = W_i + \kappa_i r K_i = H_i V_i/\mu_g, \qquad X_i = \log \mu_g + d_i,$$

where $d_i = \log(S_i/V_i) + \log(C_i^v/C_i) - \log H_i$. Here C_i^v is private variable cost, while C_i is the corresponding reported business cost. The three terms in d_i account for the difference between sales and value added, the coverage of reported costs, and the revenue wedge. If these discrepancies have mean zero within a component, its fitted mean m_g maps to the benchmark markup $\hat{\mu}_g = e^{m_g}$. Under this maintained restriction, distinct mapped markups define benchmark CES nests. The cost-ratio distri-

bution supplies a model-based grouping; the assignment analysis below examines the consequences of uncertainty about which firms compete within each group.[3]

A finite normal mixture approximates the cost-ratio distribution. The Bayesian information criterion (BIC) selects the benchmark approximation from the candidate component counts described in Appendix C. The fit imposes $\mu_g \geq 1.01$, a standard-deviation floor of 0.01, and a minimum component weight of one firm divided by the industry sample size. Components with numerically coincident means are combined when constructing the benchmark nests. Appendix C gives the candidate counts, starting values, and convergence criteria. Posterior group probabilities $\hat{a}_{ig}$ provide the benchmark fractional assignment and the nest masses in twenty sales-to-cost quantile bins per industry. These masses are sums of probabilities measured in firm-count units. Component labels are specific to an industry-year.

Input expenditure ratios supply the benchmark labor output elasticity. The first-order conditions imply

$$Z_i = \log\left(\frac{W_i}{rK_i}\right) = \log\left(\frac{\alpha_s}{1-\alpha_s}\right) + \log \kappa_i.$$

The log ratio therefore adds the common industry technology index to the firm-specific capital wedge. We estimate its location c_s with a continuous two-piece normal distribution (Wallis, 2014), allowing different dispersions on the two sides of its mode. Profiling the scales permits global optimization of the likelihood over this single location parameter. The zero-mode normalization for the log capital wedge and r_0 map this fitted mode to $\hat{\alpha}_s = \text{logit}^{-1}(c_s)$, where $\text{logit}^{-1}(x) = 1/(1+e^{-x})$. Appendix C documents the likelihood, regularization, and numerical verification.

The economic mapping permits location discrepancies:

$$\mu_g = \exp(m_g - \delta_g), \qquad \alpha_s = \text{logit}^{-1}[c_s - \nu_s - \log(r/r_0)]. \tag{17}$$

Here δ_g is the discrepancy in the component mean of the log cost ratio, and ν_s combines the economic mode of the log capital wedge with the discrepancy between the fitted location and the input-ratio mode. Capital valuation and the input discrepancy enter the same index. The reported parameter comparisons hold the baseline rental normalization at $r = r_0$ and impose $|\delta_g| \leq d_C$ and $|\nu_s| \leq d_I$, retaining $\mu_g \geq 1.01$. The two pairs $(d_C, d_I) = (0.01, 0.05)$ and $(0.025, 0.10)$ define sensitivity ranges around the benchmark mapping. A cost radius of 0.01 permits approximately one-percent multiplicative movements in μ_g, subject to its lower bound. It can produce a larger proportional movement in $\mu_g - 1$, the denominator of λ_g. The input radius is measured in log odds: because the logistic deriva-

[3]The mapping uses the static first-order conditions in Section 2. With predetermined factor payments and aggregate risk, compensation for risk can also appear as a markup in observed revenue-to-cost ratios (David et al., 2024). Such an interpretation would require the corresponding timing and risk restrictions.

tive is at most 1/4, radii of 0.05 and 0.10 permit changes in α_s of at most 0.0125 and 0.025, respectively. These ranges describe sensitivity to the economic restrictions used in the benchmark mapping.

Table 1: Sample Characteristics and Benchmark Parameter Estimates, 2007

Statistic	Value	Weight or unit
Retained firms	274,443	Firm count
Harmonized four-digit industries	425	Industry count
Benchmark CES nests	1,091	Group count
Mean labor output elasticity $\hat{\alpha}_s$	0.537	Industry value added
Median response coefficient $\hat{\lambda}_g$	2.52	Benchmark nest value added
95th percentile of $\hat{\lambda}_g$	9.14	Benchmark nest value added
Maximum $\hat{\lambda}_g$	22.07	Maximum across nests
Firms with $\max_g \hat{a}_{ig} < 0.8$	45.2%	Firm-count share

Note: Nest value added is $\sum_i \hat{a}_{ig} V_i$; industry value added sums observed V_i. The labor output elasticity uses wages and the benchmark rental normalization $r_0 = 0.2$. The response coefficient is $(1 - \hat{\alpha}_s)/(\hat{\mu}_g - 1)$. The final row reports the fraction of firms for which no group has a posterior probability of at least 0.8. Annual sample coverage, grouping, response coefficients, and input parameters appear in Tables C.1, C.2, D.12, and F.1, respectively.

Table 1 summarizes the 2007 benchmark economy. Its 1,091 fitted nests span 425 industries, with a value-added-weighted labor output elasticity of 0.537. The revenue-response coefficients vary substantially: their value-added-weighted median is 2.52, their 95th percentile is 9.14, and the maximum is 22.07. For 45.2 percent of firms, no group's posterior probability reaches 0.8. Thus both the estimated response coefficients and uncertainty about firm grouping matter when aggregating firms with different wage shares. The table weights parameter means and quantiles by value added and measures diffuse group probabilities across firms.

4.3 The Counterfactual and Its Information Sets

In the counterfactual, the capital-input wedge falls by five percent for firms whose baseline K_i/V_i strictly exceeds their four-digit industry's unweighted median. Ties are unsplit. The rule selects firms by observed capital intensity. The model's first-order condition, $rK_i/V_i = (1 - \alpha_s)H_i/(\mu_g \kappa_i)$, shows how this ratio combines technology, demand, and revenue and capital wedges. Investment dynamics can additionally reflect adjustment costs and information frictions (David and Venkateswaran, 2019). We use the ratio to define a reproducible recipient set in each annual sample, and evaluate a specified change in that set's relative capital costs. A recipient's expenditure per unit of capital services is multiplied by $0.95\, r'/r$, with the common rental rate determined in equilibrium. Wedge receipts follow the lump-sum transfer closure in Section 2. The five-percent wedge reduction corresponds to $\tau = -\log(0.95) \simeq 0.051293$. For a recipient and a nonrecipient in the same nest, their revenue ratio is multiplied by $\exp(\tau\lambda_g)$. At the median, 95th-percentile, and maximum coefficients in Table 1, these multipliers are approximately 1.138, 1.598, and 3.102. They describe relative revenue changes within

a nest, with each nest's expenditure share fixed. Their dispersion shows why the revenue-response coefficients and recipients' firm-level wage shares jointly determine the aggregate response.

The benchmark estimation supplies three inputs to the counterfactual:

$$m_{bg} = \sum_{i \in b} \hat{a}_{ig}, \qquad \hat{p}_g = \frac{\sum_i \hat{a}_{ig} V_i e_i}{\sum_i \hat{a}_{ig} V_i}, \qquad \hat{\lambda}_g = \frac{1 - \hat{\alpha}_s}{\hat{\mu}_g - 1}. \tag{18}$$

These inputs supply three distinct restrictions: m_{bg} gives nest masses in firm-count units within cost-ratio bin b; $\hat{p}_g$ is recipients' value-added share within nest g, with $e_i = 1$ for recipients and zero otherwise; and $\hat{\lambda}_g$ is the revenue-response coefficient for a nest in industry s. At benchmark coefficients, the initial assignment set fixes only the bin-level nest masses. Additional exercises fix either each industry's wage share after removing revenue and capital wedges or the nest recipient shares. Recipient-share bands allow $|p_g - \hat{p}_g| \leq h$, intersected with $[0, 1]$, while parameter comparisons vary $\hat{\lambda}_g$ through equation (17). Both the recipient shares and group markups depend on the maintained interpretation of the fitted cost-ratio components; the response coefficients also use the input-ratio estimates. The cross-survey comparison assesses sensitivity to reporting differences, while independent demand evidence would inform which exposure restrictions to maintain in an application.

Throughout the application, a *benchmark response* uses the fitted fractional assignment and benchmark parameters. Bounds over admissible nest assignments maintain the stated bin-mass and exposure restrictions. Bounds over a rectangular coefficient region additionally vary the revenue-response coefficients over their marginal intervals. Feasible outcomes under common industry technology combine assignments satisfying the stated moment restrictions with admissible markups and one common labor output elasticity within each industry. Linear programs give sharp endpoints up to the reported numerical tolerance. Nonlinear calculations report valid outer bounds, evaluated feasible outcomes, and their remaining gaps. These quantities measure economic and computational uncertainty conditional on the benchmark first stage; sampling uncertainty would additionally require a joint statistical region for its inputs.

5 Nest Composition and the Sign of Wage-Share Effects

The empirical analysis asks which information about competitors is sufficient to determine the direction of the wage-share response. We first vary group composition while keeping the observed firms, recipient rule, and response coefficients fixed. We then allow the response coefficients to vary, tracing how information about who expands and how strongly each group responds jointly determines the conclusion.

5.1 Bounds under Alternative Information Restrictions

Table 2 compares three restrictions on demand-group composition for the same 2007 firms and capital-wedge experiment. When assignments preserve only group masses within cost-ratio bins, feasible wage-share changes run from -0.0735 to $+0.7331$ percentage points, compared with a benchmark response of $+0.3619$ points. The valid outer interval is wider, $[-0.3377, 1.1173]$, because the nonlinear optimization has not closed its endpoint gaps. The two attained responses establish that alternative assignments consistent with the observed baseline and bin-level masses can reverse the effect's sign.

Table 2: Bounds on Wage-Share Effects under Alternative Information Restrictions, 2007

Information restriction	Outer bounds	Attained range
Group masses	[-0.3377, +1.1173]	[-0.0735, +0.7331]
Group masses and industry removal benchmark	[-0.3191, +1.1102]	[-0.0386, +0.7137]
Group masses and group recipient shares	[+0.0896, +0.6331]	[+0.0896, +0.6331]

Note: Wage-share changes in percentage points for the five-percent reduction in the capital wedge; 274,443 firms and benchmark response parameters. The benchmark response is +0.3619. Each row retains the same twenty-bin group masses. Row 2 fixes each industry's complete-removal benchmark. Row 3 fixes recipients' shares of group value added and uses Proposition 1; its endpoints are sharp to numerical tolerance. Rows 2 and 3 are separate information experiments. The first two rows retain valid outer bounds and exact feasible evaluations; their differences are the optimization gaps.

The second row also preserves the wage share implied by complete removal of revenue and capital wedges in every industry. Feasible finite responses still include -0.0386 and $+0.7137$ points. An industry removal benchmark averages group-specific labor shares and leaves open which firms receive the capital-wedge reduction within each group. It therefore does not determine the response to this targeted change.

The third row fixes each group's recipient share of baseline value added at the value implied by the fitted benchmark assignment. Proposition 1 then gives the sharp conditional range $[0.0896, 0.6331]$ points. Individual firm assignments and group value-added totals need not be fixed separately. Unlike the industry removal benchmark, these group recipient shares directly restrict which firms receive the capital-wedge reduction. Conditional on these shares and the benchmark response coefficients, every admissible assignment yields a positive wage-share change.

Figure 1 repeats this comparison in all ten annual samples. Benchmark responses decline from 0.823 points in 1998 to 0.362 in 2007. Group masses alone imply a positive effect in 1998 and 1999; the sign is unresolved in 2000–2003, and feasible assignments produce both signs in 2004–2007. Fixing the benchmark group recipient shares yields positive lower bounds in every year, from 0.551 points in 1998 to 0.090 in 2007. Information on recipients within groups is especially consequential in the later samples, where the group-mass restrictions permit opposing responses. Appendix D reports the annual intervals and optimization gaps.

Figure 1: Wage-Share Effects under Alternative Restrictions on Nest Composition

Note: All ten full retained annual samples, with the same five-percent reduction in the capital wedge. Gray bars give valid outer bounds under the fitted group masses, dark bars join exact feasible values, blue bars give sharp bounds over admissible nest assignments with benchmark group recipient shares fixed, and points give benchmark responses. Units are wage-share percentage points. Response parameters are held at their benchmark values.

5.2 Joint Restrictions on Composition and Parameters

Table 3 connects the group-information result to the precision of the economic mapping. Its first row repeats the fixed-share, benchmark-parameter range. The second allows each exposure share to differ from its benchmark by 2.5 percentage points. At benchmark response parameters, the outer lower bound remains positive at 0.0398 wage-share points. Exact knowledge of each share can therefore be replaced by a nonzero information tolerance in this case.

The third row fixes recipient shares and permits the smaller parameter ranges in equation (17), $(d_C, d_I) = (0.01, 0.05)$. Proposition 2 gives bounds over the rectangular coefficient region of $[0.03794, 0.79432]$ wage-share points. Because that rectangle contains the region generated by a common technology parameter in each industry, its positive lower bound covers every assignment and parameter configuration satisfying the stated moment restrictions and common industry technology within this range. Structurally feasible responses of 0.04442 and 0.79263 points show how much variation remains within this parameter region with a positive lower bound. The result determines a direction while allowing economically meaningful dispersion in the magnitude. The joint calculation

also improves on bounding parameter error separately: the uniform-error enlargement in Table B.1 gives a lower bound of -0.1387 points on the same fixed-share set, whereas the joint vertex calculation yields $+0.0379$. Accounting for assignment and parameter variation together makes the positive conclusion available.

Table 3: Wage-Share Effects under Joint Restrictions on Nest Composition and Response Parameters

Share halfwidth (pp)	(d_C, d_I)	Outer bounds	Feasible outcomes under common technology	Sign implied by the bounds
0	(0, 0)	[+0.0896, +0.6331]	[+0.0896, +0.6331]	Positive throughout
2.5	(0, 0)	[+0.0398, +0.6911]	[+0.0591, +0.6573]	Positive throughout
0	(0.01, 0.05)	[+0.0379, +0.7943]	[+0.0444, +0.7926]	Positive throughout
0	(0.025, 0.1)	[-0.0641, +1.1190]	[-0.0506, +1.1157]	Both signs feasible
2.5	(0.01, 0.05)	[-0.0382, +0.8814]	[+0.0122, +0.8275]	Unresolved

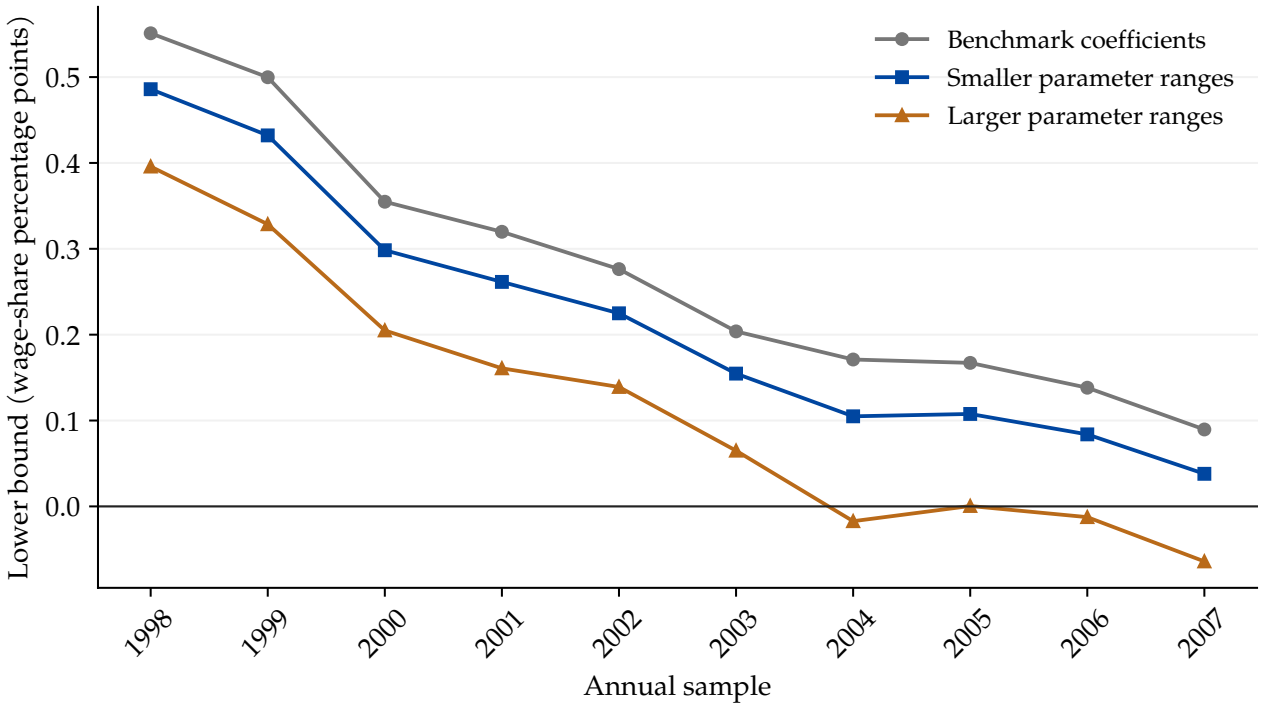
Note: This table connects the group-information result to the precision of the economic mapping using data in 2007. Wage-share changes in percentage points. Share halfwidths are percentage points of each group's recipient share of value added. (d_C, d_I) bounds the cost and input location deviations in equation (17), with $\mu_g \geq 1.01$. At fixed shares, the rectangular coefficient region has sharp LP endpoints up to numerical tolerance; Table D.3 reports the endpoint gaps, and Table D.2 reports their annual maximum. Positive-width rows use valid nonlinear outer bounds. Structurally feasible outcomes satisfy the stated assignment moments, admissible markup restrictions, and one common technology parameter per industry; they are attained outcomes within this parameter region. "Positive throughout" follows from a positive outer lower bound; "Both signs feasible" follows from attained values on both sides of zero. "Unresolved" reflects the remaining optimization gap and uncertainty from the enclosing parameter region. Appendix D gives the rectangle's attained outcomes and its numerical gaps separately.

The larger deviations, $(0.025, 0.10)$, admit a reversal. The rectangle lower bound is -0.06410 points, and an allocation with one admissible technology parameter per industry attains -0.05060 points. This admissible economy satisfying the common-technology restriction establishes that the wider primitive range permits a negative response. Response-parameter precision is economically consequential because the same recipient firms can receive different revenue weights when their groups' responsiveness changes. The firm-level wage share and recipient status then combine with those weights to determine the aggregate sign.

The last row relaxes both sources of information: recipient shares vary within 2.5 points and parameters within the smaller ranges. Its outer interval is $[-0.0382, 0.8814]$, while feasible rectangle values are $[0.0052, 0.8293]$ and structurally feasible values are $[0.0122, 0.8275]$. The lower and upper optimization gaps within the rectangular coefficient region are 0.0434 and 0.0521 points, measured against its attained rectangle values. The structurally feasible values in the main table additionally impose the common-technology restriction; their distance from the outer bounds can reflect both computational slack and the larger coefficient domain. The sign in this joint region remains unresolved at the reported computational accuracy. The two information restrictions jointly determine the scope of the positive conclusion.

Figure 2 shows that all ten smaller-range lower bounds remain positive. At the larger ranges, 2004, 2006, and 2007 admit negative responses under common industry technology. The larger-range 2005

Figure 2: Lower Bounds on Wage-Share Effects under Alternative Parameter Ranges



Note: Each annual sample fixes group recipient shares at their benchmark values. The benchmark line gives conditional allocation lower bounds. The smaller and larger ranges use $(d_C, d_I) = (0.01, 0.05)$ and $(0.025, 0.10)$, respectively, and the sharp bounds over the rectangular coefficient region. The horizontal line marks zero. The plotted bounds are for the enclosing coefficient rectangles. Negative structurally feasible responses satisfying the common-technology restriction are reported separately in Table D.2.

lower bound is only 0.00046 points, indicating a positive direction with essentially no guaranteed magnitude. The annual comparison demonstrates how the value of parameter precision varies across annual samples. Both parameter ranges and all annual results appear in Appendix D.

5.3 Nest Composition and Heterogeneous Revenue Responses

The fitted groups affect both the partition of firms and their response coefficients. A controlled 2007 comparison separates these roles. One group per industry gives a benchmark response of 0.2810 points. Retaining the fitted partition while assigning a common industry response coefficient gives 0.2701 points. Restoring heterogeneous coefficients gives 0.3619 points. Response heterogeneity therefore adds 0.0918 points relative to the common-response partition; this increment is positive in every year. The full comparison in Appendix D shows why allowing heterogeneous benchmark response coefficients can matter for a finite intervention even when a complete-removal benchmark changes little.

Nest composition also matters with common industry response coefficients: a feasible 2007 response of -0.0135 points is available, and fixing group value-added totals likewise permits a negative

response (Appendix D). Recipient shares constrain the within-nest association of wage shares and recipients directly, which explains their role in Proposition 1.

Appendix D examines finer signal bins, indivisible assignments, and revenue concentration. These diagnostics assess how the grouping restrictions and firm representation affect the reported ranges; the concentration measures describe the empirical setting for the maintained monopolistic-competition model.

6 Reporting Differences and Recipient Composition

We examine how the model's wage-share response changes with the reported cost measure and with the firms selected for the capital-wedge reduction. The first comparison uses alternative survey reports to estimate benchmark demand groups and their response coefficients while holding the selected firms fixed. The second holds each source's fitted groups and coefficients fixed and changes the selection rule. These comparisons separate sensitivity to the cost-ratio mapping from the effect of recipient composition.

6.1 Matched-Firm Comparisons across Reporting Sources

We use the 51,959 firms observed in both the industrial and tax surveys in 2007, spanning 423 harmonized four-digit industries. These firms represent 18.93 percent of firms and 40.52 percent of value added in the complete retained industrial sample. Every matched firm has positive reported employment, allowing both selection rules to be evaluated on the same sample. Industrial-survey value added, accrued wage payments, capital, and employment, along with the matched-sample estimates of industry labor output elasticities, remain fixed across reporting sources. The baseline wage share is therefore 18.4913 percent under both sources. Contemporaneous matching and symmetric source cleaning define the matched sample; Appendix E reports the selection steps and coverage.

Each reporting source supplies its own log sales-to-cost ratios and fitted mixture components, which map into benchmark nests under the restrictions in Section 4. The industrial source has 751 fitted nests and the tax source has 719. Within each source, admissible assignments preserve sums of posterior group probabilities in up to twenty cost-ratio bins per industry and fix the benchmark recipient shares p_g. This comparison holds the observed production allocation fixed while varying the reporting source used for model-based group assignment and response coefficients. It measures how reporting differences affect the conditional counterfactual under each source's maintained demand mapping.

The primary rule selects firms with industrial-survey capital relative to value added, K/V, strictly above the unweighted median of their matched-sample four-digit industry. The same 25,873 firms receive a 5 percent reduction in the model capital wedge under either reporting system. The equi-

librium rental rate remains endogenous. The alternative rule selects firms by capital per reported worker, K/N, using the same median convention and wedge reduction. It also selects 25,873 firms; N is reported headcount. For this exercise, we re-estimate the industry labor output elasticities and source-specific mixture components, and recompute the recipient thresholds, on the matched sample.

Table 4: Wage-Share Effects by Reporting Source and Recipient Rule

Recipient rule	Reporting source	Benchmark response	Conditional bounds with benchmark recipient shares
K/V	Industrial	0.3970	[0.2337, 0.5648]
K/V	Tax	0.5402	[0.3362, 0.7449]
K/N	Industrial	−0.1234	[−0.2493, 0.0129]
K/N	Tax	−0.1264	[−0.2721, 0.0310]

Note: Wage-share changes in percentage points for a 5 percent reduction in the model capital wedge. All rows use the same 51,959 firms, industrial-survey value added, wage payments, capital, and employment, and shared labor output elasticities; the baseline wage share is 18.4913 percent. Selected firms are strictly above the unweighted common-sample four-digit industry median of the stated ratio; each rule selects 25,873 firms. N denotes employment headcount. Benchmark responses use posterior group probabilities from the indicated source. Bounds optimize fractional assignments subject to that source's group masses in up to twenty cost-ratio bins per industry and benchmark recipient shares, at fixed response coefficients. Endpoints are sharp to the numerical LP tolerance; their interpretation is given in the text.

Table 4 reports a benchmark wage-share increase of 0.3970 percentage points with industrial-survey cost ratios and 0.5402 points with tax-survey cost ratios under the K/V rule, a difference of 0.1432 points. The industrial response here differs from the 0.3619-point full-sample response because the matched sample and its benchmark parameters are re-estimated. Fixing each source's fitted group recipient shares gives conditional intervals of $[0.2337, 0.5648]$ and $[0.3362, 0.7449]$ points. The positive sign holds in both source-specific conditional sets, while the magnitude changes appreciably with the reported costs used for grouping. Baseline value added and wage payments, as well as the selected firms, are common across the two calculations.

Selecting by K/N instead yields benchmark wage-share changes of -0.1234 points with industrial costs and -0.1264 with tax costs. The corresponding conditional intervals, $[-0.2493, 0.0129]$ and $[-0.2721, 0.0310]$, contain both signs. The two sources give similar benchmark responses under this rule, but the fitted recipient shares do not establish the sign across all admissible assignments. Which firms are selected, and how their wage shares compare with those of other firms in their nests, are therefore central to the result.

6.2 Recipient Composition and the Ranking of Wage-Share Effects

Panel A of Table 5 describes the firms selected by each rule. Both select 25,873 of the 51,959 matched firms. Under K/V, selected firms account for 48.11 percent of matched-sample value added and have a wage share of 22.57 percent, compared with 14.71 percent among other firms. Under K/N, selected firms account for 75.67 percent of value added and have a lower wage share, 16.81 percent, than the

remaining firms, 23.72 percent. Average wage payments per worker are higher among firms selected by K/N than among those selected by K/V, with indices of 115.35 and 99.24. Their value added per worker also differs substantially, with indices of 126.87 and 81.30. The two rules select equal numbers of firms but different parts of the production distribution. The per-worker indices set the matched-sample aggregate to 100; subgroup wage shares are ratios of total wages to total value added.

Table 5: Recipient Characteristics and Within-Nest Contributions

Panel A. Recipient Characteristics in the Common Baseline Economy

Rule	Status	Firms	VA share (%)	W/V (%)	W/N index	V/N index
K/V	Nonrecipient	26,086	51.89	14.71	101.10	127.11
K/V	Recipient	25,873	48.11	22.57	99.24	81.30
K/N	Nonrecipient	26,086	24.33	23.72	77.31	60.28
K/N	Recipient	25,873	75.67	16.81	115.35	126.87

Panel B. Exact Within-Nest Contributions at the Benchmark Assignment

Rule	Reporting source	Positive terms	Negative terms	Net response
K/V	Industrial	0.4020	−0.0050	0.3970
K/V	Tax	0.5442	−0.0041	0.5402
K/N	Industrial	0.0288	−0.1523	−0.1234
K/N	Tax	0.0535	−0.1798	−0.1264

Note: Panel A uses industrial reports on all 51,959 common firms. Ratios are ratios of subgroup totals; per-worker indices equal 100 at the full common-sample aggregate, and employment is reported headcount. Panel B sums $k_g(C_g - p_g L_g)/T$ separately over positive and negative terms at each source's benchmark assignment; units are wage-share percentage points. Net responses are calculated before rounding. Appendix E reports the recipient cross-classification, group counts, and accounting checks.

The identities

$$\frac{K}{V} = \frac{K/N}{V/N}, \qquad \frac{W}{V} = \frac{W/N}{V/N}$$

explain how these profiles can diverge. Capital relative to value added reflects both capital per worker and value added per worker. A firm can therefore have high K/N and high wages per worker while devoting a relatively small share of value added to wages. The overlap comparison makes the change concrete. A total of 14,370 firms switch status: 7,185 selected only by K/V account for 3.13 percent of value added and have a wage share of 45.52 percent, whereas 7,185 selected only by K/N account for 30.70 percent of value added and have a wage share of 10.72 percent. Equal recipient counts consequently conceal a large change in the economic weight and wage shares of the firms receiving the wedge reduction.

Panel B links these aggregate profiles to the exact within-nest calculation in equation (15). Each nest's contribution is the recipients' excess wage bill $C_g - p_g L_g$, multiplied by the nonnegative finite-change reweighting factor k_g and divided by total value added T. The excess is measured relative to recipients' share of the nest's value added: a positive term means that recipients account for more

of the nest's wage bill than of its value added. A negative term means that their wage-bill share is lower. The model combines this within-nest association with the revenue-response coefficient to determine the aggregate change. Panel A describes the overall recipient sample, and Panel B measures the associations within benchmark nests that enter the counterfactual.

Under K/V, positive contributions sum to 0.4020 points and negative contributions to -0.0050 with industrial-survey costs; the corresponding tax-survey totals are 0.5442 and -0.0041. Under K/N, positive contributions are 0.0288 and 0.0535 points, while negative contributions are -0.1523 and -0.1798. Their sums reproduce the four benchmark responses, up to rounding. A lower capital wedge reallocates revenue toward selected firms within nests, so the sign of each contribution depends on their wage shares relative to their nest peers. The distinct selection rules place much more economic weight on positive contributions under K/V and on negative contributions under K/N. The optimization over admissible assignments in Table 4 establishes positive conditional bounds under K/V and bounds containing both signs under K/N.

Information on recipient composition also allows the two interventions to be ranked. Within each reporting source, maintain the benchmark recipient-share vector for each rule on the common set of within-bin group masses and hold the source's benchmark response coefficients fixed. The source's benchmark assignment satisfies both vectors, so the resulting common set is nonempty. On this set, the wage-share effect of the K/V rule exceeds that of the K/N rule by at least 0.2207 percentage points using industrial reports and 0.3051 points using tax reports. These conservatively rounded lower bounds follow from the separate response bounds; Appendix E gives their derivation. The comparison concerns the same 5 percent proportional reduction in capital wedges under each rule; the recipient capital and value-added totals differ. Nest composition information can therefore determine the ranking of wage-share effects even when the sign of one effect remains unresolved.

7 Conclusion

A reduction in selected firms' relative capital costs can shift revenue shares toward those firms and away from their competitors. Whether this reallocation raises the aggregate wage share depends on the wage shares of the firms on each side of that shift and on the strength of substitution within their demand groups. We show how to evaluate this channel when enterprise data leave the relevant grouping uncertain. Recipients' initial shares of group value added, together with group masses and revenue-response coefficients, make the exact finite effect linear in unknown assignments. This result characterizes the range of wage-share responses consistent with the specified information about competitors.

In the Chinese manufacturing application, a standardized five-percent reduction in selected firms' capital wedges gives a positive benchmark wage-share effect. Assignments consistent with the bench-

mark cost-signal group masses can imply either sign in later years. Holding recipients' group value-added shares fixed gives a positive conditional lower bound in every annual sample. This conclusion also survives smaller parameter deviations, while wider admissible deviations permit a negative response under common industry technology. With both recipient shares and parameters varying, the reported bounds leave the sign unresolved. The matched-firm comparison shows that an alternative rule selecting firms by capital per worker reverses the benchmark prediction. Under fixed recipient shares and benchmark response coefficients, the capital-to-value-added rule still has the larger effect under both reporting systems.

The analysis identifies two useful targets for further evidence: which close substitutes compete with the firms receiving capital-cost relief, and how strongly revenue responds within those groups. Information on recipients' economic weight within groups can determine the direction or ranking of a finite wage-share response even when individual demand-group assignments remain uncertain.

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Online Appendix

A Theory and Proofs

A.1 Proportional interventions and the revenue response operator

The main model has two primary factors and physical constant returns. For this extension, let $q_i = A_i \prod_{j=1}^{J} x_{ij}^{\theta_{jg}}$ within a fixed known group, and define

$$b_{jg} = \theta_{jg}/\mu_g > 0, \qquad B_g = \sum_j b_{jg} < 1, \qquad \lambda_{jg} = \frac{b_{jg}}{1 - B_g}. \tag{A.1}$$

The J inputs are primary factors in fixed positive aggregate supply. Firms face common competitive prices v_j, positive factor-cost wedges c_{ij}, and positive revenue-retention wedges H_i. The group partition, upper-level expenditure shares, technology, and active firms remain fixed. All net wedge receipts are returned lump sum. There are no fixed resource requirements, entry costs, or intermediate-input supply links in this statement. The first-order conditions are

$$c_{ij} v_j x_{ij} = b_{jg} H_i V_i. \tag{A.2}$$

These revenue elasticities determine the specified nominal response; physical output elasticities retain their separate meaning in production and markup measurement (Bond et al., 2021).

Proposition A.1 (Information for proportional interventions)**.** *Let $\widehat{H}_i = H_i'/H_i > 0$ and $\hat{c}_{ij} = c_{ij}'/c_{ij} > 0$ be specified intervention ratios and put $\zeta_{ij} = \log \widehat{H}_i - \log \hat{c}_{ij}$. If $f_{ij} = v_j x_{ij}/V_i$, the nominal counterfactual satisfies*

$$\omega_i' = \frac{\omega_i \exp(\lambda_g^\top \zeta_i)}{\sum_{h \in g} \omega_h \exp(\lambda_g^\top \zeta_h)}, \quad f_{ij}' = f_{ij} \frac{\widehat{H}_i}{\hat{c}_{ij}}, \quad FS_j' = \sum_g \beta_g \sum_{i \in g} \omega_i' f_{ij}'. \tag{A.3}$$

Let $\mathcal{U}_g$ span the within-group contrasts $\zeta_i - \zeta_h$ permitted by an intervention class. Two response vectors generate the same entire relative-revenue response operator if and only if their difference lies in $\mathcal{U}_g^\perp$. If $\mathcal{U}_g = \mathbb{R}^J$, this operator identifies $b_{jg} = \lambda_{jg}/(1 + \sum_k \lambda_{kg})$.

Proof. Writing the firm's revenue function as $V_i = D_g A_i^{1/\mu_g} \prod_j x_{ij}^{b_{jg}}$, substitute (A.2) to obtain

$$V_i^{1-B_g} = D_g A_i^{1/\mu_g} H_i^{B_g} \prod_j \left(\frac{b_{jg}}{c_{ij} v_j} \right)^{b_{jg}}.$$

Common demand and factor-price terms cancel from within-group ratios. Taking state ratios leaves $\widehat{H}_i^{B_g/(1-B_g)} \prod_j \hat{c}_{ij}^{-b_{jg}/(1-B_g)} = \exp(\lambda_g^\top \zeta_i)$. Normalizing yields the first formula, while ratios of the first-order conditions give the payment-intensity formula. Aggregating payments gives FS_j'. For positive-revenue pairs,

$$\log(\omega_i'/\omega_h') - \log(\omega_i/\omega_h) = \lambda_g^\top (\zeta_i - \zeta_h).$$

Equality of operators is therefore equivalent to orthogonality to every permitted contrast. Equality of odds and normalization gives the converse. Summing $\lambda_{jg} = b_{jg}/(1 - B_g)$ yields the inverse mapping. □

The necessity statement concerns the full response operator. A particular aggregate share can depend on fewer combinations. For smooth paths, put $s_{ij} = d \log H_i/d\tau|_0 - d \log c_{ij}/d\tau|_0$. Differentiating the weighted average gives

$$\left.\frac{dFS_j}{d\tau}\right|_0 = \sum_g \beta_g \left\{ E_g[f_j s_j] + \sum_k \lambda_{kg} \operatorname{Cov}_g(f_j, s_k) \right\}. \tag{A.4}$$

The first term changes payment intensity within firms; the second reallocates revenue. The proposition specifies the response parameters required for an intervention class.

A.2 Exact capital response and fixed exposure information

Compatibility with the observed baseline. The retained observations have $V_i, W_i, K_i > 0$. Fix positive baseline factor prices w, r. For any nonzero assignment a_{ig}, give the type value added $a_{ig}V_i$, wages $a_{ig}W_i$, capital $a_{ig}K_i$, and labor $a_{ig}W_i/w$. Set

$$H_{ig} = \frac{\mu_g W_i}{\alpha_s V_i}, \qquad \kappa_{ig} = \frac{(1-\alpha_s)W_i}{\alpha_s r K_i}.$$

These positive wedges satisfy both input first-order conditions. CES prices and quantities can be chosen to reproduce the positive within-group revenue weights, and productivity can then be chosen to reproduce each type's quantity at its assigned inputs. Aggregate inputs equal the observed totals, and $\beta_g = M_g/T$ reproduces group expenditures. Revenue wedges and productivity are calibrated for each admissible economy and held fixed during its intervention. The capital wedge above is common across copies of a record within its industry. This construction supplies the baseline economies whose finite responses are compared below.

For the capital intervention, $s_{iL} = 0$ and $s_{iK} = e_i$, while physical constant returns give $B_g = 1/\mu_g$. Proposition A.1 yields (7) and (8). In a fractional economy, baseline within-group weights are $a_{ig}V_i/M_g$, so the post-intervention mean firm-level wage share is Q_g^W/Q_g^V. Multiplication by the fixed-within-economy share M_g/T proves (14). Fixed positive group count masses and $V_i > 0$ bound every denominator away from zero.

For binary exposure, the exact change can also be written

$$\Delta F_\tau(a) = \frac{1}{T}\sum_g M_g \left\{ \frac{L_g + \gamma_g C_g}{M_g + \gamma_g U_g} - \frac{L_g}{M_g} \right\}, \qquad \gamma_g = \exp(\tau\lambda_g) - 1. \tag{A.5}$$

Differentiating at zero gives

$$D(a) = \left.\frac{dF_\tau(a)}{d\tau}\right|_0 = \frac{1}{T}\sum_g \lambda_g \left(C_g - \frac{L_g U_g}{M_g} \right). \tag{A.6}$$

When both exposure types occur, the recipient and nonrecipient wage means are unchanged by the tilt within their respective types. Their changing mixture weight is $p_g(\tau)$, proving (10). Uniform exposure or constant firm-level wage share within a group gives zero response.

Details for Proposition 1. With fixed p_g, substituting $U_g = p_g M_g$ gives the equivalent level formula

$$F_\tau(a) = \frac{1}{T} \sum_{ig} a_{ig} W_i \frac{1 + \gamma_g e_i}{1 + \gamma_g p_g}. \tag{A.7}$$

The denominator is positive because $\gamma_g > -1$ and $p_g \in [0,1]$. All group share equalities must be retained: with heterogeneous p_g, the last group's restriction need not follow from the others. Compactness and linearity deliver attained sharp endpoints for the declared fractional set. For general exposure a supplied ratio $\chi_g = M_g / Q_g^V$ similarly yields the linear objective $T^{-1} \sum_g \chi_g Q_g^W$ and linear restrictions $M_g - \chi_g Q_g^V = 0$. This ratio is specific to the shock. Binary p_g determines it at every shock through $1 + \gamma_g p_g$.

If e_i takes finitely many values $e_1, \ldots, e_J$, category shares $p_{gj} = \sum_i a_{ig} V_i 1\{e_i = e_j\} / M_g$ give

$$Q_g^V / M_g = \sum_j p_{gj} \exp(\tau \lambda_g e_j).$$

The restrictions are linear, and $J - 1$ shares per group are independent before other restrictions are imposed. A mean exposure generally does not determine this exponential moment. The sufficient-information ordering can also collapse in particular datasets. With two groups and $p_1 \neq p_2$, observed totals imply $M_1 = (\sum_i V_i e_i - p_2 T)/(p_1 - p_2)$, so exposure shares themselves determine group value added in that special case.

A.3 Coefficient Regions, Centering, and Common Technology

For fixed shares, subtracting L_g / T from (A.7) gives (15). For any predetermined group constant c_g, the fixed-share restriction also gives

$$\mathcal{R}_g(a) = \sum_i a_{ig} (W_i - c_g V_i)(e_i - p_g). \tag{A.8}$$

Centering at the observed industry wage share preserves the objective and the allocation set.

Details for Proposition 2. For a fixed allocation, monotonicity of k_g selects a coefficient endpoint according to the sign of $\mathcal{R}_g$. The finite vertex minimum commutes with minimization over the compact allocation set. If industry restrictions are independent, the aggregate optimum is the sum of industry optima with observed value-added weights. A verified sign for $\mathcal{R}_g$ throughout the allocation set fixes its coefficient endpoint in advance. This can reduce enumeration while leaving its worst-case exponential size unchanged.

Let λ^0 lie in the rectangle and define

$$d_g^k = \max\{|k_g(\lambda_g^-, p_g) - k_g(\lambda_g^0, p_g)|, |k_g(\lambda_g^+, p_g) - k_g(\lambda_g^0, p_g)|\}.$$

An LP supplies the uniform bound

$$B_k = \frac{1}{T} \max_{a \in \Lambda(p)} \sum_{ig} a_{ig} |W_i - c_g V_i| \, |e_i - p_g| \, d_g^k. \tag{A.9}$$

The triangle inequality applied to (A.8) gives $|\Delta F(a; \lambda, p) - \Delta F(a; \lambda^0, p)| \leq B_k$ uniformly. Thus $[L_0 - B_k, U_0 + B_k]$ encloses the joint range whenever $[L_0, U_0]$ encloses the benchmark range. This bound

uses the actual assignment constraints and vanishes when industry wages are proportional to value added. A valid upper bound for its maximization supplies the guarantee.

For exposure bands, use the actual share $p = U/M$ at each allocation. The group change is $T^{-1}\gamma(CM - LU)/(M + \gamma U)$, and its derivative is

$$\frac{1}{T}\frac{\tau \exp(\tau\lambda)M(CM - LU)}{(M + \gamma U)^2}.$$

Its sign is independent of λ, establishing the endpoint reduction for any compact allocation restrictions independent of the coefficients. The allocation problem at each vertex remains nonlinear when shares vary.

Shared technology. At the baseline rental normalization, the empirical parameter regions use

$$\alpha_s = \text{logit}^{-1}(c_s - \nu_s), \quad |\nu_s| \le d_I, \qquad \mu_g \in [\max\{1.01, \mu_g^0 e^{-d_C}\}, \mu_g^0 e^{d_C}].$$

The deviation in the input-ratio normalization, ν_s, includes economic and statistical location discrepancies; the radii and floor are maintained sensitivity assumptions. If the induced technology interval is $[\alpha_s^-, \alpha_s^+]$, the marginal coefficient limits are

$$\lambda_g^- = (1 - \alpha_s^+)/(\mu_g^+ - 1), \qquad \lambda_g^+ = (1 - \alpha_s^-)/(\mu_g^- - 1).$$

Their Cartesian product covers the common-α_s region. An outcome feasible under common industry technology combines one admissible α_s for all groups in an industry with admissible markup parameters and an assignment satisfying the stated moment restrictions. Its exact evaluation supplies an inner value. The technology endpoints supply feasible evaluations; opposing group contributions can also create an interior optimum. Computing sharp bounds over the common-technology region requires outer coverage of that region. An industry consisting of one unexposed firm contributes zero and stays in the aggregate denominator.

A.4 A Fixed Removal Benchmark with Opposite Responses

There are six records, each with value added one, wages $(0.1, 0.5, 0.9, 0.2, 0.6, 1.0)$, and exposure $(1, 1, 1, 0, 0, 0)$. Two groups have labor output elasticity 0.5 and markup parameters $(1.5, 1.25)$, giving response coefficients $(1, 2)$. Allocation A assigns group labels $(1, 1, 2, 1, 2, 2)$; allocation B assigns $(1, 2, 1, 2, 2, 1)$. Both have group masses and value added $(3, 3)$, exposed value added $(2, 1)$, and aggregate wage share 0.55. Their removal wage share is $\frac{1}{2}(0.5/1.5 + 0.5/1.25) = 0.366\overline{6}$.

In A, group wages are $(0.8, 2.5)$ and exposed wages $(0.6, 0.9)$; in B they are $(2.0, 1.3)$ and $(1.0, 0.5)$. Equation (A.6) gives derivatives $+1/30$ and $-1/30$. For 5 percent relief, the exact responses are $+0.17236$ and -0.16662 percentage points. This two-group, one-bin example preserves exposure shares as well as the removal benchmark. Fixed exposure information therefore makes the target linear while allowing its sign to depend on the observed wage-exposure association.

A.5 Nominal equivalence and the removal accounting benchmark

The revenue-response formulation also permits a precise nominal equivalence. Write $\rho_g = 1/\mu_g \in (0, 1)$ for the CES revenue exponent.

Proposition A.2 (A power transformation preserving revenue elasticities)**.** *In the primary-factor economy of Proposition A.1, fix the partition, group expenditure shares, factor supplies, and numerical wedge schedules. For $h_g > \rho_g$, replace A_i by $A_i^{h_g}$, θ_{jg} by $h_g\theta_{jg}$, and μ_g by $h_g\mu_g$. Every interior equilibrium has a corresponding equilibrium with identical firm revenues, factor inputs and prices, profits, transfers, and nominal factor-income shares. The correspondence holds for each specified positive wedge intervention.*

Proof. Set $\tilde{q}_i = q_i^{h_g}$, $\tilde{\rho}_g = \rho_g/h_g$, and $\tilde{p}_i = p_i q_i^{1-h_g}$. The common demand shifter $D_g = E_g/\sum_{i\in g} q_i^{\rho_g}$ is unchanged. At every input bundle,

$$\widetilde{V}_i = D_g\left(A_i^{h_g}\prod_j x_{ij}^{h_g\theta_{jg}}\right)^{\rho_g/h_g} = D_g A_i^{\rho_g}\prod_j x_{ij}^{b_{jg}} = V_i.$$

The profit problems, factor clearing, and household budget coincide. Transformed CES demand satisfies $\tilde{p}_i\tilde{q}_i = V_i$ and $\tilde{q}_i^{\tilde{\rho}_g} = q_i^{\rho_g}$, so all equilibrium conditions hold. Powers $1/h_g$ give the inverse correspondence. □

The transformation preserves b_{jg} and changes physical curvature and preferences. Physical output and price-index-based real wages can therefore differ. Common technology across an industry's groups requires a common power across those groups. Imposing constant returns to scale fixes this power ambiguity. The correspondence concerns specified numerical wedge changes; interventions targeting the markup itself need not be preserved. It complements the identification analysis of Chow et al. (2026) by stating the nominal implication within this maintained economy.

Baseline payment ratios still require economic restrictions to determine the response coefficients. For any positive two-factor revenue elasticities summing to less than one, the choices

$$H_i = \frac{W_i}{b_{Lg}V_i}, \qquad \kappa_i = \frac{b_{Kg}H_iV_i}{rK_i} \tag{A.10}$$

match both observed payment first-order conditions. Choosing positive quantities with CES revenue weights proportional to V_i and then selecting productivity to produce them rationalizes the baseline. Concavity from $B_g < 1$ verifies optimality. Thus unrestricted firm wedges can support different revenue elasticities on the same baseline accounts.

After removing both revenue and capital wedges, the two-factor constant-returns model gives

$$LS^{\text{rem}} = \sum_g \beta_g\frac{\alpha_s}{\mu_g}, \qquad KS^{\text{rem}} = \sum_g \beta_g\frac{1-\alpha_s}{\mu_g}, \qquad PS^{\text{rem}} = \sum_g \beta_g\left(1-\frac{1}{\mu_g}\right).$$

Government net receipts vanish and these shares sum to one. Relative to a pooled industry parameter μ_s^P, the grouped wage-share increment is

$$LS^{\text{rem}}_{\text{groups}} - LS^{\text{rem}}_{\text{pooled}} = \sum_s \beta_s\alpha_s\left(\sum_{g\subset s}\frac{\beta_{g|s}}{\mu_g} - \frac{1}{\mu_s^P}\right). \tag{A.11}$$

The target-equivalent parameter

$$\mu_s^T = \left(\sum_{g\subset s}\beta_{g|s}/\mu_g\right)^{-1} \tag{A.12}$$

compresses the removal wage-share target, while heterogeneous finite responses continue to depend on exposure and group-specific coefficients.

B Computation and Numerical Accuracy

B.1 Direct finite-target outer bounds

Fix the observed records, exposure, finite shock, and response coefficients. The allocation set is nonempty and compact, satisfies the row and bin–group margins in (12), and can include further linear moment restrictions. Assume $V_i > 0$, $W_i \geq 0$, and positive total count mass for each group. Define $q_{ig}, M_g, Q_g^V, Q_g^W$ as in Section 3.

Proposition B.1 (Direct finite bounds and computational accuracy)**.** *For valid boxes on $x_g = M_g/Q_g^V$ and $y_g = Q_g^W/Q_g^V$, replace each $M_g Q_g^W/Q_g^V$ by z_g, and impose the four perspective McCormick inequalities below together with the linear ratio bounds. The minimum and maximum of $T^{-1}\sum_g z_g$ over this LP enclose the exact finite-target range. At every relaxed feasible point,*

$$\left| F_\tau(a) - \frac{1}{T}\sum_g z_g \right| \leq \frac{1}{4T}\sum_g Q_g^V(a)(x_g^+ - x_g^-)(y_g^+ - y_g^-). \tag{B.1}$$

The right side is linear in allocation, so maximizing it yields a uniform error bound. Domain subdivisions covering the original ratio boxes preserve valid outer bounds, and exact evaluations at feasible allocations supply inner values.

Proof. For each group, write $R = Q_g^V$, $N = Q_g^W$, $M = M_g$, $x = M/R$ and $y = N/R$. The relaxation imposes

$$\begin{aligned} z &\geq x^- N + y^- M - x^- y^- R, & z &\geq x^+ N + y^+ M - x^+ y^+ R, \\ z &\leq x^+ N + y^- M - x^+ y^- R, & z &\leq x^- N + y^+ M - x^- y^+ R, \end{aligned}$$

and $x^- R \leq M \leq x^+ R$, $y^- R \leq N \leq y^+ R$. Every original allocation embeds at $z = MN/R = Rxy$: the inequalities follow from the four nonnegative products formed by distances from the endpoints of the x, y box. Hence minimization and maximization of the relaxed linear objective give outer bounds. On the unit square, the distance between xy and any point between its McCormick lower and upper envelopes is at most $1/4$. Rescaling and multiplying by R/T proves (B.1). Since R is linear in a, maximizing the sum of these bounds is another LP. This gives a uniform approximation bound. Joint attainability of group errors is governed by the shared allocation constraints. □

The implementation normalizes $\sum_i V_i = 1$. On its nonnegative-wage support it may additionally impose $0 \leq z_g \leq \max_i(W_i/V_i)$, because $z_g = M_g$ times a tilted group wage share and $M_g \leq 1$. This finite box is also used in the numerical dual restoration below.

Ratio bounds and additional information. With only row and bin–group masses, a group-specific ratio $\sum a_i Z_i / \sum a_i D_i$, $D_i > 0$, is bounded by roots of

$$\min_{0 \leq a_i \leq 1,\, \sum_{i \in b} a_i = m_{bg}} \sum_i a_i (Z_i - c D_i),$$

and the analogous maximum. A minimum selects the smallest scores within each bin, with a fractional boundary record if needed; the maximum reverses the ordering. The selected value is strictly decreasing through its unique zero because the total group mass is positive and denominators are bounded away from zero. The remaining group quotas can fill the residual row capacities. Apply this with $(Z, D) = (V, Vq_g)$ and (Wq_g, Vq_g). Retaining the outward root brackets gives valid coordinate boxes. After additional moment restrictions are imposed, these original boxes remain conservative outer boxes, but need no longer equal the exact coordinate ranges of the smaller set. That distinction is preserved in the implementation.

Feasible search on the exact target. The analytic gradient is

$$\frac{\partial[TF_\tau(a)]}{\partial a_{ig}} = V_i \frac{Q_g^W}{Q_g^V} + \frac{M_g W_i q_{ig}}{Q_g^V} - \frac{M_g Q_g^W V_i q_{ig}}{(Q_g^V)^2}. \tag{B.2}$$

It follows by differentiating the three linear moments. Linear minimization or maximization of this gradient uses the *full* transportation and additional-information constraints. A convex line segment between feasible allocations preserves every linear equality and inequality. The implementation searches the exact finite objective along that segment using a fixed grid and bounded one-dimensional refinement, always retaining the zero step. This feasible improvement method supplies attained values for the nonconvex target while preserving all economic-moment restrictions. Accuracy is assessed through the outer–inner computational gap. For exact feasible endpoint allocations, their connecting segment lies in the same convex constraint set. The finite objective is continuous along that segment because every denominator stays positive. Consequently every value between their two objective values is attained somewhere on the segment. This connectedness result applies to fractional allocations. Indivisible assignments define a discrete feasible set. Saved numerical witnesses are evaluated at the reported feasibility tolerances.

Domain coverage and bounds. An initial ratio box covers every feasible allocation. Splitting one ratio interval at its midpoint yields two children whose union covers the parent, including their common boundary. Each child retains the original constraints and its own linear ratio bounds. The minimum of the active lower LP bounds and maximum of the active upper LP bounds remain outer bounds. Candidate allocations from any node are evaluated under the exact original target; feasible refinement may leave that node's box while remaining in the original admissible set, which is sufficient for an inner value. For separable industries, aggregate endpoints and gaps are the observed-value-added-weighted sums of the corresponding industry quantities. A cross-industry constraint couples industries and requires a joint optimization.

For the primary unrestricted-exposure annual calculations, the algorithm initially permits at most sixteen feasible steps per direction and industry. It then prioritizes the industry contributing most to an aggregate endpoint gap, and a group box with a large perspective-envelope error. Each split retains both children. The common annual limits are 512 child LP pairs and a twenty-minute wall-time safety cap. Each child supplies two LP starting allocations, with at most four feasible-refinement steps per direction from each start (eight per direction per child). Selection and stopping follow these accuracy and resource rules for both directions. Unresolved gaps and numerical statuses are reported. The common stopping target is an aggregate gap of at most 0.02 percentage points in each direction, subject to these resource limits.

Numerical dual checks and unresolved solves. For minimization of $c^\top x$ subject to $Ax = b$, $Gx \le d$ and finite box $l \le x \le u$, any y and $z \le 0$ imply

$$c^\top x \ge b^\top y + d^\top z + \sum_k \min(l_k r_k, u_k r_k), \qquad r = c - A^\top y - G^\top z.$$

This follows from $z^\top Gx \ge z^\top d$ and the coordinate box minima. The bound therefore allows restoration of dual stationarity residuals. The implementation records primal residuals and an explicit outward numerical tolerance adjustment. To check a supposedly infeasible child, it minimizes a common nonnegative slack on its scaled inequalities while retaining its equalities and variable box. A positive restored dual lower bound excludes zero slack. Without a verified exclusion, the child's parent bound is retained conservatively. An unresolved root LP retains the outer range implied by observed firm-level wage shares; an unresolved feasible-search oracle retains existing feasible allocations. No industry is removed because of a numerical status. These analytical bounds are evaluated in double precision with explicit outward numerical tolerance adjustments; formal directed-rounding certificates are outside this implementation.

B.2 Exposure bands and coefficient vertices

The monotonicity result in Appendix A.3 reduces a Cartesian product of response-coefficient intervals to its vertices even when exposure shares vary. For a given vertex and bands $p_g(a) \in [p_g^-, p_g^+]$, define

$$f_e(\lambda, p) = \frac{(\exp(\tau\lambda) - 1)(e - p)}{1 + p(\exp(\tau\lambda) - 1)}.$$

For $\gamma = \exp(\tau\lambda) - 1 \ge 0$ and binary e,

$$\frac{\partial f_e}{\partial p} = -\frac{\gamma(1 + \gamma e)}{(1 + \gamma p)^2} \le 0.$$

Let $d_{ig} = W_i - c_g V_i$, where c_g is fixed. The centered identity (A.8) holds at the actual share $p_g(a)$, so the finite change is $T^{-1} \sum_{ig} a_{ig} d_{ig} f_{e_i}(\lambda_g, p_g(a))$. Taking the minimum and maximum of each coefficient $d_{ig} f_{e_i}(\lambda_g, p)$ over the two band endpoints gives lower and upper linear objectives; a negative d_{ig} reverses their ordering. Optimizing these objectives under the share-band constraints provides valid outer bounds. Zero-width bands recover the exact fixed-share LP.

Taking the extreme bounds across every coefficient vertex covers the joint rectangle. Positive-width coefficient envelopes can leave an optimization gap. Exact evaluations at their feasible optimizing allocations and subsequent feasible improvements supply attained values. The implementation reports those values alongside the outer bounds. With benchmark shares, the preliminary centered bound (A.9) is an LP; its restored upper dual value supplies the uniform guarantee.

The benchmark-parameter exposure-band calculations use the direct finite relaxation strengthened by the exposure-share ratio bounds, with sixteen feasible improvement steps and no subdivision. Their reported gaps reflect that fixed budget. The joint coefficient-and-share-band calculations use the centered coefficient envelopes at every vertex, retaining scalar certificates and feasible assignments. Searches subject to common industry technology evaluate a parameter grid and bounded scalar refinements at saved assignments. The resulting structurally feasible outcomes provide inner values; the outer bounds retain the full rectangular coefficient region.

Table B.1: The centered finite-parameter error bound

Year	B_k, smaller	Lower after enlargement	B_k, larger	Lower after enlargement
1998	0.2123	+0.3388	0.6524	-0.1014
1999	0.2102	+0.2896	0.6554	-0.1556
2000	0.2109	+0.1439	0.6653	-0.3105
2001	0.2311	+0.0887	0.7518	-0.4320
2002	0.2056	+0.0707	0.6349	-0.3585
2003	0.2289	-0.0251	0.7573	-0.5535
2004	0.3153	-0.1443	0.9441	-0.7730
2005	0.2735	-0.1063	0.8318	-0.6647
2006	0.2675	-0.1293	0.7783	-0.6401
2007	0.2283	-0.1387	0.6987	-0.6091

Note: Effects are wage-share percentage points. The restored upper dual bound for the centered error LP enlarges the fixed-share benchmark interval. A nonpositive enlarged lower endpoint is inconclusive, as the tighter vertex calculations in Table D.2 demonstrate. The same two declared primitive radii and all annual retained records are used.

B.3 Linear dual bounds and information restrictions

At fixed p and λ, define $c_{ig} = T^{-1}W_i k_g(\lambda_g, p_g)(e_i - p_g)$. The equality restrictions are row masses, bin–group masses, and $U_g - p_g M_g = 0$. Any multipliers u_i, v_{bg}, z_g satisfying

$$u_i + v_{b(i)g} + z_g V_i(e_i - p_g) \le c_{ig}$$

give the lower bound $\sum_i u_i + \sum_{bg} m_{bg} v_{bg}$. This is ordinary LP duality, with numerical residuals restored using the implied boxes $0 \le a_{ig} \le 1$. A positive restored bound excludes a negative response throughout the declared information set.

Changing exposure information changes both the objective and the constraint matrix, and dual solutions can be nonunique. We evaluate the value of additional information by imposing or relaxing the restrictions and recomputing the range. If information sets are nested, any valid outer bound for a larger set also bounds a smaller set. Intersecting such bounds industry by industry preserves validity and implements the nesting used in the empirical information tables.

If only selected industries have fixed shares, add their sharp LP endpoints to valid bounds for the remaining industries, using observed value-added weights. The aggregate endpoints are outer bounds until every contributing endpoint is sharp. For a uniform share radius, define

$$\mathcal{A}(h) = \{a \in \mathcal{A} : |U_g - p_g^0 M_g| \le h M_g \text{ for all } g\}, \qquad h_*(q) = \inf\{h \ge 0 : \exists a \in \mathcal{A}(h),\ \Delta F(a) \le q\}.$$

An outer lower bound strictly above q excludes breakdown at that radius. A feasible response at most q supplies an upper bracket using its actual maximum group-share discrepancy. A lower outer bound below q without a corresponding witness leaves that radius unresolved. The empirical thresholds $q = 0$ and $q = 0.0005$ correspond to zero and a 0.05-percentage-point gain; the latter is an illustrative economic magnitude. This conditional breakdown calculation parallels the robustness question in Masten and Poirier (2020).

B.4 Local bounds and finite Taylor control

The direct finite objective can be checked against a separate valid bound obtained from its derivative. The following results preserve the local and Taylor calculations used in that comparison.

Proposition B.2 (Local perspective relaxation)**.** *Let $V_i > 0$, every group have positive total count mass, and $\lambda_g \geq 0$. For valid boxes $x_g^- \leq L_g/M_g \leq x_g^+$ and $y_g^- \leq U_g/M_g \leq y_g^+$, introduce z_g satisfying*

$$\begin{aligned} z_g &\geq x_g^- U_g + y_g^- L_g - x_g^- y_g^- M_g, & z_g &\geq x_g^+ U_g + y_g^+ L_g - x_g^+ y_g^+ M_g, \\ z_g &\leq x_g^+ U_g + y_g^- L_g - x_g^+ y_g^- M_g, & z_g &\leq x_g^- U_g + y_g^+ L_g - x_g^- y_g^+ M_g. \end{aligned} \tag{B.3}$$

The minimum and maximum of $T^{-1}\sum_g \lambda_g(C_g - z_g)$ enclose the range of D in (A.6). Every interval between two feasible exact values is attained in the fractional allocation set.

Proof. Each original allocation embeds at $z_g = L_g U_g/M_g$ in the perspective McCormick inequalities. This proves outer validity. Positive group masses bound the denominators away from zero, so D is continuous on the compact convex allocation polytope. The segment joining any two allocations therefore supplies all intermediate values. □

Proposition B.3 (Local relaxation error)**.** *At any relaxed feasible point, with objective $\widetilde{D}$,*

$$|D(a) - \widetilde{D}(a,z)| \leq \frac{1}{4T}\sum_g \lambda_g M_g (x_g^+ - x_g^-)(y_g^+ - y_g^-). \tag{B.4}$$

The maximum of the right side is a linear-programming uniform bound.

Proof. The distance between a bilinear function and either of its box envelopes is at most one quarter of the product of the side lengths. Multiplication by $M_g\lambda_g/T$ and summation proves (B.4). Since $M_g = \sum_i a_{ig} V_i$, the resulting bound is linear in allocation. □

Coordinate mean boxes follow from the ratio-root construction above, using $(Z, D) = (W, V)$ and (Ve, V). The local gradient is

$$\frac{\partial[TD(a)]}{\partial a_{ig}} = \lambda_g V_i \left(\ell_i - \frac{L_g}{M_g}\right)\left(e_i - \frac{U_g}{M_g}\right).$$

Proposition B.4 (Uniform finite Taylor control)**.** *For binary exposure and any admissible allocation, let $\Delta\ell_s$ denote the observed firm-level wage-share range in industry s. Then*

$$|\Delta F_\tau(a) - \tau D(a)| \leq \frac{\tau^2}{12\sqrt{3}\,T}\sum_g M_g \lambda_g^2 \Delta\ell_{s(g)}. \tag{B.5}$$

Maximizing the right side gives a uniform linear-programming bound. For $\tau \geq 0$, adding and subtracting that value from τ times the local outer endpoints gives a finite outer interval.

Proof. For a group with both exposure types, its wage-mean change is $d_g[p_g(\tau) - p_g]$. Its second derivative is $\lambda_g^2 d_g p_g(\tau)[1 - p_g(\tau)][1 - 2p_g(\tau)]$. Use $|d_g| \leq \Delta\ell_{s(g)}$ and $\max_{p\in[0,1]} p(1-p)|1-2p| = 1/(6\sqrt{3})$ in Taylor's theorem, then weight by M_g/T and sum. Uniformly exposed groups contribute zero. □

Both the direct and Taylor intervals cover the same finite target, so their intersection remains valid. The perspective envelopes follow McCormick (1976); the economic reductions identify the group moments and assignment restrictions to which those envelopes are applied.

Table B.2: Numerical accuracy and resource limits

Year	Child LP pairs	Seconds	Max. residual	Tolerance met
1998	512	224	8.03e-10	No
1999	512	447	9.67e-10	No
2000	512	630	9.81e-10	No
2001	512	645	8.00e-10	No
2002	512	537	9.21e-10	No
2003	512	573	9.26e-10	No
2004	512	765	9.45e-10	No
2005	512	715	9.74e-10	No
2006	512	788	7.80e-10	No
2007	512	1139	9.91e-10	No

Note: The target is at most 0.02 wage-share percentage points per aggregate endpoint. The deterministic cap is 512 child LP pairs per year, with a 1,200-second safety stop checked between operations. Retained outer bounds cover unresolved numerical branches. Residuals concern scaled linear constraints; full primal/dual and split checks are in the supplement.

B.5 Numerical diagnostics

All primary years use the same retained-record target, exposure rule, tolerance, and resource limits. Unresolved solver statuses retain conservative bounds and existing feasible values. The computational supplement includes synthetic identity checks and a separate reconstruction of saved allocations. Annual economic ranges, concentration, and the indivisible-record comparison are reported in Appendix D.

C Data and Benchmark Parameter Construction

This appendix documents the annual samples and benchmark inputs used in Section 4 and in every finite-counterfactual calculation.

C.1 Fields, eligibility, and economic coverage

The industrial-enterprise source extract used here jointly observes the required production, intermediate-expenditure, wage, sales, and business-cost fields in 1998–2007. It contains no 1988–1993 records; the 1994–1997 records lack the common core fields, and the intermediate-expenditure field needed for the primary value-added measure is unavailable after 2007. These availability statements concern the source extract used here. Each annual analysis uses its complete retained sample, without requiring survival across years. Table C.1 gives annual counts and economic coverage. The tax reports are matched separately for the comparison in Appendix E.

Table C.1 records the same screening sequence for all ten years. Retained samples increase from 112,707 firms in 1998 to 274,443 in 2007, while the retention rate stays close to 93 percent. The value-added, wage, and capital columns show which economic activity the tail treatment removes. In 2007, the retained firms account for 91.2 percent of eligible value added, 93.1 percent of wages, and 91.8 percent of capital. The counterfactual denominator and every benchmark estimate use these retained annual samples.

We first remove exact duplicates in the extracted production, identity, and input records. Raw four-digit industrial codes must lie in sectors 06–46. Legal identifiers and names are Unicode-normalized, whitespace is removed, and identifier hyphens are removed. Names and identifiers

Table C.1: Annual Sample Coverage under Uniform Tail Treatment

Year	Eligible firms	Retained firms	Firms (%)	Value added (%)	Wages (%)	Capital (%)
1998	121,245	112,707	93.0	93.5	95.0	95.4
1999	121,273	112,808	93.0	93.6	95.2	95.6
2000	124,493	115,724	93.0	92.5	95.0	94.8
2001	135,097	125,592	93.0	93.0	90.4	94.3
2002	144,541	134,447	93.0	93.2	94.7	94.3
2003	164,131	152,595	93.0	92.8	94.8	93.8
2004	237,526	220,686	92.9	90.9	93.6	92.2
2005	234,998	218,358	92.9	91.0	93.1	93.2
2006	262,673	243,969	92.9	91.2	93.5	92.6
2007	295,593	274,443	92.8	91.2	93.1	91.8

Note: The primary sample removes observations outside the first and 99th percentiles of any of four log ratios within year and harmonized two-digit industry: wages/output, rental capital/output, value added/output, and sales/business cost. All cutoffs are computed once on the eligible sample. Percentages refer to that sample; coverage of the industrial core appears in the replication files.

must be nonempty; identifiers must contain 8–18 alphanumeric characters and cannot consist entirely of zeros or nines or equal an explicit invalid code. All records for an identifier that remains duplicated within a year are excluded. The initial core then requires finite positive gross output Q, intermediate expenditure M, wages W, and sales S, with $Q > M$ and $0.5 \leq Q/S \leq 2$.

The structural sample additionally requires positive finite supplied capital, positive paired business cost, and a unique harmonized manufacturing industry. The existing concordance maps `cic02` through 2002 and `cic03` thereafter to `cic_adj`. Its source and checksum are recorded in the replication manifest. Requiring a manufacturing mapping excludes mining, utilities, and unmapped records. Normalized identifiers, names, and industry definitions follow the linkage considerations discussed by Brandt et al. (2014).

Current-price gross output is `gyzczxjxgd`, intermediate expenditure is `zjtrhj`, wages are `bnyfgzze`, accrued employee welfare benefits are `bnyfflfze`, and employment is `cyrs`. Primary value added is $V = Q - M$. Reported and processed value added (`gyzjz` and `va`) and VAT (`bnyjzzs`) are retained as distinct variables. Their agreement with $Q - M + VAT$ is a reporting diagnostic; the primary denominator remains $Q - M$.

Sales and costs are paired by their source labels: `cpxssr/cpxscb` in 1998–2000 and 2002–2003, `zysr/zycb` in 2001 and 2006–2007, and `zyywsr/zycb` in 2004–2005. The five early product-cost years observe product-sales costs but lack `zycb`. Selling, management, financial, depreciation, and interest expenses remain separate. The capital input is `rk1`; average net book capital, `gdzcjznpjye`, supplies the alternative capital measure in Appendix F. Available checks link much of the supplied series to its investment and depreciation recursion. Its full initial-stock and price-unit history remains unavailable, so rental coefficients are expressed per supplied capital unit.

Wages are the primary payment measure. The $W + B$ comparison requires finite nonnegative benefits on the same firms for both measures. Missing or negative supplements remain unavailable. Employment restrictions apply only when constructing employment or per-worker statistics; firms with unavailable employment remain in the primary wage/value-added and structural samples. These definitions measure recorded wages and specified supplements.

C.2 Uniform tail treatment

Within each year and harmonized two-digit industry, we form $\log(W/Q)$, $\log(0.2K/Q)$, $\log(V/Q)$, and $\log(S/C)$ on the same eligible sample. Linear-interpolated empirical 0.01 and 0.99 quantiles are computed once. The primary mask is the intersection of the four closed intervals. Every retained firm enters estimation and aggregation; there is no subsequent screen based on component size, posterior probability, likelihood improvement, or counterfactual weight. Annual retained samples contain 112,707–274,443 firms and retain approximately 91–94 percent of eligible value added.

The winsorized comparison clips the same ratios at the same cutoffs on every eligible firm. For the corresponding clipping operators,

$$\widetilde{W} = Qe^{\mathrm{clip}_W(\log(W/Q))}, \qquad \widetilde{K} = (Q/0.2)e^{\mathrm{clip}_K(\log(0.2K/Q))}, \qquad \widetilde{V} = Qe^{\mathrm{clip}_V(\log(V/Q))},$$

$$\widetilde{M} = Q - \widetilde{V}, \qquad \widetilde{C} = S/e^{\mathrm{clip}_C(\log(S/C))}.$$

Output and sales stay fixed, and recorded benefits are unchanged. Original levels and all exclusions remain in the audit. The replication files provide cutoff values and count, output, sales, value-added, wage, and capital coverage relative to both eligible manufacturing and the initial industrial core. Ratio selection changes the population to which the economic location restrictions apply. A normal mixture fitted after trimming approximates the selected distribution; winsorization additionally creates boundary mass points.

C.3 The Log Sales-to-Cost Ratio and Benchmark Nests

Private variable cost in the value-added model satisfies $C_i^v = W_i + \kappa_i r K_i = H_i V_i/\mu_g$. With paired reported business cost C_i, the observed signal therefore obeys

$$X_i = \log(S_i/C_i) = \log\mu_g + d_i, \qquad d_i = \log(S_i/V_i) + \log(C_i^v/C_i) - \log H_i. \tag{C.1}$$

The benchmark mapping sets the mean of d_i in each component distribution to zero. This restriction connects a fitted component mean to the value-added demand parameter while allowing individual reporting discrepancies and revenue wedges.

Within each four-digit industry-year, the benchmark density is

$$f_s(x) = \sum_{g=1}^{G_s} \pi_{sg}\,\sigma_{sg}^{-1}\phi((x - m_{sg})/\sigma_{sg}), \qquad \sum_g \pi_{sg} = 1. \tag{C.2}$$

Under correct finite-normal-mixture specification, distinct component distributions identify the parameters up to relabeling (Teicher, 1963). Interpreting distinct component means as CES nests additionally requires the economic restriction in (C.1) and one nest for each distinct CES parameter. If the component mean of d_i is unrestricted, $m_{sg} = \log\mu_{sg} + \delta_{sg}$ leaves μ_{sg} unidentified. The benchmark uses $\mu_{sg} = e^{m_{sg}}$ and $\epsilon_{sg} = \mu_{sg}/(\mu_{sg} - 1)$. Exponentiating the fitted component mean implements this restriction directly.

A gross-output cost ratio requires this boundary mapping even in a Cobb–Douglas technology. With intermediate-input elasticity η, labor output elasticity $\alpha_s(1-\eta)$, gross revenue R_i, and undistorted intermediate spending, the first-order conditions imply

$$\frac{W_i}{R_i - M_i} = \frac{\alpha_s(1-\eta)H_i}{\mu^{GO} - \eta H_i}.$$

Writing the expression as $\alpha_s H_i/\mu^{VA}$ gives $\mu^{VA} = (\mu^{GO} - \eta H_i)/(1-\eta)$, which depends on the revenue wedge. The empirical value-added model thus imposes its own output and cost boundaries through (C.1).

David and Venkateswaran (2019) use materials expenditure shares to discipline markup dispersion under a common materials elasticity and materials choices with no input-specific wedge. In the gross-output environment just described, the corresponding condition is $M_i/R_i = \eta H_i/\mu^{GO}$. With common η and $H_i = 1$, variation in this share measures relative markups; their level additionally depends on η. Applying this source of information to the present value-added model requires the output-boundary conversion above and restrictions on H_i. Moreover, distinct product groups can have the same markup, so markup information must be paired with a restriction on which firms compete. These requirements specify how production information can complement the demand-group restrictions used in the counterfactual.

Candidate component counts run from one to $\min\{20, \max(1, \lfloor n_s/10 \rfloor)\}$. The constraints are $m_{sg} \geq \log(1.01)$, $\sigma_{sg} \geq 0.01$, and $\pi_{sg} \geq 1/n_s$. The variance floor prevents likelihood spikes; the location floor implies $\mu \geq 1.01$ and $\epsilon \leq 101$. The one-component model is analytic. Higher counts use four predetermined starts and a bounded continuation of the best start. Among converged monotone solutions we retain the highest likelihood, then select the count by BIC with nominal parameter count $3G_s - 1$. This regularized multistart procedure supplies the benchmark approximation, with every candidate and convergence flag retained.

The E-step uses log-sum-exp normalization. The M-step projects the weighted signal mean onto the location bound, projects the residual standard deviation onto its floor, and solves the weight simplex with the common lower bound by active-set normalization. These operations and selected posterior calculations are independently checked. For the mixture fit, numerical certification concerns convergence at the selected local solution.

The benchmark posterior is

$$r_{ig} = \frac{\widehat{\pi}_g \widehat{\sigma}_g^{-1} \phi((X_i - \widehat{m}_g)/\widehat{\sigma}_g)}{\sum_h \widehat{\pi}_h \widehat{\sigma}_h^{-1} \phi((X_i - \widehat{m}_h)/\widehat{\sigma}_h)}. \tag{C.3}$$

Components with numerically coincident means are combined before constructing the benchmark nests, with their posterior probabilities added. Component labels are local to each industry-year. A modal assignment selects the largest combined posterior probability; the counterfactual benchmark uses the full fractional assignment.

C.4 Input location and numerical construction

The relative-input first-order condition is

$$Z_i = \log[W_i/(r_0 K_i)] = \operatorname{logit}(\alpha_s) + \log \kappa_i, \qquad r_0 = 0.2. \tag{C.4}$$

The benchmark assigns zero mode to the log relative-input wedge. It fits a continuous two-piece normal distribution with location c and positive scales s_-, s_+:

$$f(z) = \frac{\sqrt{2/\pi}}{s_- + s_+} \begin{cases} e^{-(z-c)^2/(2s_-^2)}, & z < c, \\ e^{-(z-c)^2/(2s_+^2)}, & z \geq c. \end{cases} \tag{C.5}$$

Continuity links the left probability to $s_-/(s_- + s_+)$ (Wallis, 2014). For positive scales the mode and the two curvatures identify this statistical family. The economic interpretation of its location uses (C.4).

Table C.2: Mixture Components and Posterior Nest Membership Probabilities

Year	Components	Benchmark nests	Maximum posterior below 0.8 (%)	Unavailable candidates	Unavailable $\hat{\alpha}$ VA (%)
1998	740	740	38.0	9	0.00000
1999	766	766	37.5	6	0.00000
2000	822	822	39.5	3	0.00062
2001	833	833	41.5	1	0.00035
2002	845	845	38.9	6	0.00000
2003	962	962	40.9	7	0.00000
2004	1012	1012	49.5	7	0.00000
2005	1031	1031	44.1	8	0.00000
2006	1062	1062	44.0	12	0.00000
2007	1091	1091	45.2	13	0.00000

Note: Diffuse posterior membership means a maximum probability below 0.8. Components with coincident means are combined when constructing benchmark nests. Unavailable candidates failed the convergence criteria; all selected candidates converged. The final column reports the value-added share of constant input-ratio samples with no finite input-location estimate. Their contribution is propagated using $\alpha \in [0, 1]$.

Let $S_-(c) = \sum_i (c - Z_i)_+^2$, $S_+(c) = \sum_i (Z_i - c)_+^2$, and $T(c) = S_-(c)^{1/3} + S_+(c)^{1/3}$. Scale first-order conditions give

$$\hat{s}_j(c) = S_j(c)^{1/3}\sqrt{T(c)/n_s}, \qquad \ell_p(c) = n_s\left[\log\sqrt{2/\pi} + \tfrac{1}{2}\log n_s - \tfrac{1}{2} - \tfrac{3}{2}\log T(c)\right]. \tag{C.6}$$

The finite-sample problem minimizes T on the observed range. A nonconstant sample has $T(c)^3 \geq \sum_i (Z_i - \bar{Z})^2 > 0$, so the likelihood supremum is finite. At a boundary one scale may approach zero; we retain the corresponding half-normal supremum. A constant sample allows both scales to collapse and has no finite likelihood supremum.

Global interval bounds use the concavity of $z^{1/3}$. On a center interval, a secant lower bound for each cube root yields $T(c) \geq a_- S_-(c) + a_+ S_+(c) + b$ with nonnegative a_-, a_+. The right side is convex; sorted prefix sums locate its minimizer. Subdivision maintains a feasible T_U and a lower bound T_L, yielding the likelihood-gap bound

$$0 \leq \sup_c \ell_p(c) - \ell_p(\hat{c}) \leq \frac{3n_s}{2}\log(T_U/T_L).$$

The implementation normalizes the observed range, recomputes residuals near cancellation, subtracts a floating-point allowance from lower bounds, and retains unresolved intervals. The requested per-observation gap is 10^{-9}. Available extended precision on the estimation platform has a 53-bit significand. Numerical bounds use these analytic inequalities and floating-point safeguards.

Across the trimmed and winsorized wage specifications, six industry-year fits stop at arithmetic resolution just above that tolerance: industry 1810 in trimmed 2006–2007 and winsorized 2004–2007. Their saved near-optimal-center hulls imply at most 0.000112 percentage points of movement in a pooled, modal, or posterior linear removal benchmark; the trimmed maximum is 0.000096. These flags remain in the output. Constant-input industries receive $\alpha_s \in [0, 1]$ in linear benchmark aggregation. The unavailable single-record input cases have uniform exposure and zero finite counterfactual response, and remain in the aggregate denominator.

C.5 Cost-Ratio Bins, Nest Masses, and Interpretation

The primary assignment set uses 20 within-industry quantile bins of X, keeping ties together. For each bin b and benchmark nest g, the mass in firm-count units is $m_{sbg} = \sum_{i\in(s,b)} r_{ig}$. Admissible assignments satisfy

$$a_{ig} \geq 0, \qquad \sum_g a_{ig} = 1, \qquad \sum_{i\in(s,b)} a_{ig} = m_{sbg}.$$

The posterior is feasible by construction. Fractional assignments preserve these sums of membership probabilities while allowing nest membership to depend on value added, the firm-level wage share, and intervention exposure. Each admissible assignment calibrates its own baseline nest expenditure shares from assigned value added and holds those shares fixed during its counterfactual.

These constraints specify the information carried from the first-stage fit into the counterfactual. They preserve nest masses in each bin while allowing the assignment of individual cost-ratio values within a bin to change. A nest's raw weighted mean and other within-component distributional features can therefore vary across admissible assignments. The fitted distributions construct the benchmark parameters and bin masses; the bounds condition on these masses and the stated exposure and parameter restrictions. An exact zero residual sample moment would be an additional assignment restriction, distinct from the distributional zero-mean normalization used to construct benchmark markups. Regularization can also separate fitted component means from posterior-weighted sample means.

Benchmark exposure shares are computed from the same posterior,

$$p_g^0 = \frac{\sum_i r_{ig} V_i e_i}{\sum_i r_{ig} V_i},$$

where e_i identifies firms strictly above the annual industry median K_i/V_i. The information exercises fix these shares at their benchmark values or allow stated bands around them. Nested 10–20–40–80 bin comparisons preserve ties and refine the bin-specific nest-mass restrictions.

The economic locations also admit explicit discrepancies. Write the component mean of the log cost ratio as $m_g = \log \mu_g + \delta_g$. If η_s is the economic log-wedge mode and ξ_s is the difference between the fitted input location and the selected-population mode, define $\nu_s = \eta_s + \xi_s$. The mapping is

$$\mu_g = e^{m_g - \delta_g}, \qquad \alpha_s = \text{logit}^{-1}[c_s - \nu_s - \log(r/r_0)]. \tag{C.7}$$

A bound on $|\nu_s|$ restricts the sum of economic and fitting discrepancies. The radii specify the permitted economic deviations in addition to the fitted cost and input distributions. Appendix F gives the resulting response-coefficient ranges and capital-measurement checks.

The benchmark construction supplies the inputs to the allocation and parameter sets defined in the main text. All candidate statuses, bin masses, and economic-location choices are retained alongside the reported results.

D Complete Finite-Counterfactual Results

This appendix reports the full annual results and information diagnostics supporting Section 5.

D.1 Annual allocation and coefficient ranges

Table D.1 combines the original-mass calculations with the exact fixed-exposure-share bounds for every annual sample. The original-mass lower feasible values are negative in 2004–2007. Fixed benchmark exposure shares yield positive lower bounds in every year. Records with zero responses remain in the aggregate value-added denominator.

Table D.1: Complete annual finite responses and group exposure information

Year	Benchmark	Feasible values	Outer bounds	Fixed-p bounds	Lower gap	Upper gap
1998	0.823	[0.275, 1.227]	[0.065, 1.531]	[0.551, 1.082]	0.210	0.305
1999	0.784	[0.259, 1.171]	[0.035, 1.470]	[0.500, 1.042]	0.224	0.299
2000	0.654	[0.149, 1.055]	[-0.077, 1.380]	[0.355, 0.943]	0.226	0.325
2001	0.637	[0.107, 1.074]	[-0.125, 1.427]	[0.320, 0.939]	0.232	0.353
2002	0.539	[0.094, 0.892]	[-0.101, 1.189]	[0.276, 0.794]	0.195	0.297
2003	0.500	[0.041, 0.889]	[-0.163, 1.182]	[0.204, 0.790]	0.203	0.293
2004	0.528	[-0.038, 1.009]	[-0.348, 1.602]	[0.171, 0.879]	0.310	0.593
2005	0.488	[-0.019, 0.934]	[-0.295, 1.364]	[0.167, 0.802]	0.276	0.430
2006	0.442	[-0.039, 0.840]	[-0.317, 1.251]	[0.138, 0.735]	0.278	0.411
2007	0.362	[-0.073, 0.733]	[-0.338, 1.117]	[0.090, 0.633]	0.264	0.384

Note: All ten complete retained annual samples in Table C.1; 5% capital-wedge relief for firms strictly above the annual industry median K/V, benchmark coefficients, and 20-bin group masses. All entries are wage-share percentage points. Feasible values and outer bounds use the original mass restrictions; the outer bounds intersect valid direct and Taylor bounds. Gaps compare their corresponding endpoints. Fixed-p bounds add benchmark group exposure shares and solve the exact allocation LP. The common nonlinear optimization target is 0.02 points per endpoint; unresolved gaps remain reported.

Table D.2: Joint response-coefficient rectangles in every annual sample

Year	Smaller rectangle	Feasible lower value (common technology)	Larger rectangle	Feasible lower value (common technology)
1998	[+0.4859, +1.2410]	+0.4891	[+0.3958, +1.5703]	+0.4028
1999	[+0.4321, +1.2022]	+0.4358	[+0.3286, +1.5450]	+0.3363
2000	[+0.2984, +1.0975]	+0.3028	[+0.2049, +1.4382]	+0.2152
2001	[+0.2615, +1.1127]	+0.2664	[+0.1608, +1.5145]	+0.1717
2002	[+0.2249, +0.9371]	+0.2292	[+0.1391, +1.2426]	+0.1486
2003	[+0.1547, +0.9540]	+0.1591	[+0.0649, +1.3483]	+0.0750
2004	[+0.1049, +1.1123]	+0.1112	[-0.0172, +1.5968]	-0.0034
2005	[+0.1077, +1.0022]	+0.1136	[+0.0005, +1.4129]	+0.0127
2006	[+0.0839, +0.9272]	+0.0896	[-0.0125, +1.3011]	-0.0002
2007	[+0.0379, +0.7943]	+0.0444	[-0.0641, +1.1190]	-0.0506

Note: Effects are wage-share percentage points. Smaller radii are $(d_C, d_I) = (0.01, 0.05)$; larger radii are $(0.025, 0.10)$. Every row uses the full same-year retained sample, fixed benchmark group exposure shares, original 20-bin group masses and the common 5% reduction in the capital wedge. Each rectangle contains the corresponding common-technology parameter set. The structurally feasible columns report lower attained responses under one technology parameter per industry; their upper counterparts are retained in the saved output. Across all twenty annual parameter cases, the largest rectangle endpoint gap is 2.36×10^{-9} wage-share percentage points (below 2×10^{-9} in 2007). Every declared year and both radii are reported.

Table D.2 reports both parameter radii in every year. The smaller coefficient rectangles retain positive lower bounds throughout. The larger rectangles contain negative structurally feasible responses in 2004, 2006, and 2007. The 2005 larger-rectangle lower bound is positive but close to zero (0.00046

points before table rounding). Structurally feasible values satisfy the stated assignment moments, use one technology parameter per industry, and have admissible group cost locations; their construction provides feasible responses within this common-technology set. The rectangle bounds cover that set, with exact LP extrema for the larger coefficient domain. The joint information experiment in Table 3 additionally allows 2.5-point exposure-share bands with the smaller parameter radii. Its outer lower bound is −0.03823 wage-share points and its lowest attained rectangle value is +0.00522, leaving the direction unresolved.

Table D.3 reports the complete 2007 precision comparison, including the attained values in the rectangular coefficient region and their endpoint gaps. These values distinguish the remaining numerical uncertainty within the rectangle from the additional restriction that one technology parameter must apply to all groups in an industry. The joint-band lower gap is 0.04344 wage-share points; the distance from its outer lower bound to its lowest structurally feasible value is 0.05040 points. The latter can reflect both computational slack and the larger coefficient domain.

Table D.3: Complete exposure and parameter precision comparison, 2007

Share band halfwidth (pp)	(d_C, d_I)	Outer bounds	Attained values in rectangle	Feasible outcomes (common technology)	Rectangle gaps lower / upper
0	(0, 0)	[+0.0896, +0.6331]	[+0.0896, +0.6331]	[+0.0896, +0.6331]	1.1e-09 / 8.1e-10
2.5	(0, 0)	[+0.0398, +0.6911]	[+0.0591, +0.6573]	[+0.0591, +0.6573]	0.0194 / 0.0338
0	(0.01, 0.05)	[+0.0379, +0.7943]	[+0.0379, +0.7943]	[+0.0444, +0.7926]	9.4e-10 / 7.9e-10
0	(0.025, 0.1)	[-0.0641, +1.1190]	[-0.0641, +1.1190]	[-0.0506, +1.1157]	1.1e-09 / 1.1e-09
2.5	(0.01, 0.05)	[-0.0382, +0.8814]	[+0.0052, +0.8293]	[+0.0122, +0.8275]	0.0434 / 0.0521

Note: All effects and computational gaps are wage-share percentage points. The share-band halfwidth is measured in percentage points of recipients' group value-added share. The gaps compare the outer bounds with attained values in the same coefficient rectangle. The structurally feasible column imposes a common technology parameter within each industry. At benchmark coefficients the rectangle and structurally feasible columns coincide. With fixed exposure shares, the rectangle endpoints are sharp up to the reported LP gaps; the common-technology column reports attained outcomes. With a positive share-band halfwidth, the objective is nonlinear and the reported outer and attained values leave the displayed computational gaps. This table preserves all five information conditions in Table 3.

D.2 Which group statistics change the bounds?

Table D.4 gives the full 2007 comparison with the industry removal benchmark, group value added, and group wages. The first four rows form nested restrictions, so every smaller set inherits valid outer bounds from its supersets industry by industry. Fixed exposure shares form a separate comparison: fixing M_g and fixing p_g are generally nonnested, while fixing both M_g and exposed value added U_g fixes $p_g = U_g/M_g$.

Tables D.5 and D.6 add benchmark value-added and wage moments to exposure information. The binary-exposure objectives remain linear under the displayed restrictions. Finer nested signal bins progressively restrict the association of records and groups while preserving the benchmark posterior. The tables report each bin resolution and annual sample.

Table D.7 separates the posterior partition from heterogeneous response coefficients. In 2007, the benchmark response is 0.281 points with one group per industry, 0.270 with the posterior partition and a common industry response, and 0.362 with the fitted group responses. The comparison holds firms, payments, exposure, and industry input parameters fixed.

Table D.4: Economic restrictions and a controlled response comparison, 2007

Information	Outer interval (pp)	Attained interval (pp)	Lower gap	Upper gap
Original masses	[-0.3377, 1.1173]	[-0.0735, 0.7331]	0.2642	0.3842
Industry removal benchmark	[-0.3191, 1.1102]	[-0.0386, 0.7137]	0.2805	0.3965
Group value added	[-0.3118, 1.0939]	[-0.0022, 0.6844]	0.3097	0.4095
Group value added and wages	[-0.2965, 1.0610]	[0.1152, 0.6280]	0.4117	0.4330
Common industry λ	[-0.204, 0.830]	[-0.014, 0.424]	0.190	0.406

Note: Exact finite wage-share changes in percentage points; 5% reduction in the capital wedge and all 274,443 records. For the first four rows, valid outer bounds are intersected industry by industry along the nested information sets; original attained values are unchanged. Group value added fixes the industry removal benchmark at the maintained coefficients. The common-response row changes coefficients, so its bounds are not intersected with the other rows. Remaining gaps measure computation, not sampling uncertainty.

Table D.5: Finite responses under alternative group information

Year	Fix M_g, U_g	Fix M_g, L_g, U_g
1998	[0.560, 1.076]	[0.644, 1.020]
1999	[0.515, 1.034]	[0.595, 0.985]
2000	[0.365, 0.933]	[0.456, 0.867]
2001	[0.334, 0.923]	[0.428, 0.855]
2002	[0.285, 0.789]	[0.358, 0.735]
2003	[0.225, 0.782]	[0.306, 0.727]
2004	[0.194, 0.862]	[0.301, 0.773]
2005	[0.195, 0.784]	[0.278, 0.704]
2006	[0.161, 0.721]	[0.240, 0.658]
2007	[0.109, 0.619]	[0.174, 0.554]

Note: All retained annual observations; 20 common quantile bins. Entries are changes in the aggregate wage share in percentage points after a 5% reduction in the capital wedge for the fixed exposed records. M_g, L_g, and U_g denote group value added, wages, and exposed value added. Each information restriction fixes the indicated moment at its benchmark-model value. With binary exposure each displayed problem is a linear program, solved over fractional assignments. Restored dual bounds are reported, with outward numerical tolerance adjustments and residual checks; assignments represent a continuum of observed firm types.

Table D.6: Nested grouping precision and finite-response bounds, 2007

Requested bins	Fix U_g/M_g	Fix M_g, U_g	Fix M_g, L_g, U_g
10	[0.076, 0.652]	[0.095, 0.636]	[0.163, 0.563]
20	[0.090, 0.633]	[0.109, 0.619]	[0.174, 0.554]
40	[0.105, 0.611]	[0.124, 0.600]	[0.184, 0.543]
80	[0.126, 0.589]	[0.143, 0.580]	[0.196, 0.530]

Note: The same 274,443 records and posterior assignments are used in every row. Quantile bins are nested and ties remain unsplit; realized counts may be smaller. Each bin fixes group masses measured as sums of posterior membership probabilities in firm-count units. Units and economic restrictions are as in Table D.5.

Table D.7: Separating partition and response heterogeneity

Year	One industry group	Partition, common λ_s	Partition, fitted λ_g
1998	0.722	0.713	0.823
1999	0.689	0.679	0.784
2000	0.562	0.545	0.654
2001	0.531	0.520	0.637
2002	0.466	0.452	0.539
2003	0.414	0.385	0.500
2004	0.411	0.396	0.528
2005	0.362	0.354	0.488
2006	0.328	0.318	0.442
2007	0.281	0.270	0.362

Note: Posterior finite responses to the same 5% reduction, in percentage points. The first column uses one group per industry. The second preserves the fitted posterior partition but assigns every group the industry response $\lambda_s = (1-\alpha_s)/(\mu_s^{\text{pooled}} - 1)$, with $\mu_s^{\text{pooled}} = \exp\{\max(\overline{X}_s, \log 1.01)\}$. The third restores heterogeneous fitted responses. Firms, wages, value added, exposure and α_s are unchanged. The comparison separates the benchmark partition from the response coefficients.

Table D.8: How precise must group exposure information be?

Exposure-share information	Outer interval	Attained interval	Lower gap	Upper gap
Exact share	[0.090, 0.633]	[0.090, 0.633]	0.000	0.000
±1 pp	[0.064, 0.659]	[0.070, 0.647]	0.006	0.012
±2.5 pp	[0.040, 0.691]	[0.059, 0.657]	0.019	0.034

Note: All 274,443 retained 2007 records, the same 20-bin count restrictions, and a 5% reduction in the capital wedge. The first row fixes every posterior exposure share p_g. The other rows allow $|U_g - p_g M_g| \leq hM_g$, independently for each group; group expenditure and wage totals are not supplied separately. "pp" in the first column refers to exposure-share percentage points; all interval entries refer to wage-share percentage-point changes. Band rows use a direct finite relaxation strengthened by the known exposure-share ratio bounds, with sixteen feasible improvement steps and no subdivision. Their outer lower bounds are positive even though extrema are not computed sharply. The radii are hypothetical information tolerances.

D.3 The amount and precision of exposure information

The 2007 fixed-share experiment concerns 1,091 nest-level exposure shares in 425 industries. Optimizing individual shares over the original assignment set leaves 40 ranges narrower than 10^{-6}; these groups account for 0.68 percent of benchmark value added. Other shares generally add information beyond the signal-bin masses. The vector of individual ranges need not be jointly attainable.

Table D.9 orders industries by the improvement in their valid lower bound when all their group shares are fixed. The displayed set of 87 industries covers 63.00 percent of value added and yields a lower bound of 0.00028 points; 142 industries cover 77.04 percent and yield 0.05007 points. This data-dependent ordering supplies sufficient information sets within the observed sample. All firms remain in each calculation. Combining bounds and feasible values places the increase in the true minimum from fixing every share between 0.16306 and 0.42730 points; the larger change in valid lower bounds also includes improved numerical tightness.

Table D.10 reports every computed share halfwidth. At four share percentage points, the lower bound remains positive. Eight- and sixteen-point bands leave the sign unresolved because the lower bound is negative and the lowest attained response is positive. A negative feasible value occurs at a 32-point band. Allowing numerical feasibility tolerance, the smallest halfwidth permitting a nonpositive

Table D.9: How much group exposure information is sufficient?

Industries constrained	Nests constrained	VA covered (%)	Lower bound (pp)	Attained lower (pp)
0	0	0.00	-0.33770	-0.07347
25	92	36.26	-0.13649	0.01557
50	175	50.80	-0.06087	0.03917
87	299	63.00	0.00028	0.05799
100	338	66.77	0.01622	0.06284
142	460	77.04	0.05007	0.07472
200	624	85.64	0.07229	0.08251
425	1091	100.00	0.08959	0.08959

Note: All 274,443 records remain in every calculation. Industries are ordered by their computed improvement in the valid lower bound when all their group exposure shares are fixed. Selected industries use the exact fixed-share LP; the others retain the baseline outer bound. Thus aggregate lower bounds are valid but generally not sharp. This data-dependent ordering gives sufficient observed sets, without a minimality claim. VA coverage is the share of value added in industries receiving the restrictions. The group count measures how many shares are fixed; restrictions may be linearly dependent. The 87-industry lower bound is positive but economically small.

Table D.10: Information precision and the sign of the finite response

Share halfwidth (exposure-share pp)	Valid lower bound (wage-share pp)	Attained lower (wage-share pp)
0.00	0.08959	0.08959
1.00	0.06410	0.07032
1.75	0.05112	0.06383
2.50	0.03978	0.05914
4.00	0.01942	0.05242
8.00	-0.02939	0.03774
16.00	-0.11909	0.00384
32.00	-0.25595	-0.05269

Note: 2007, all 274,443 records, benchmark response coefficients and 20-bin group masses. The share halfwidth is the permitted deviation of each group's exposed VA share from its benchmark, clipped to $[0, 1]$. The lower bounds are valid outer bounds; attained values are exact finite evaluations at feasible allocations. A negative lower bound with a positive attained value leaves the sign unresolved. Thresholds of zero and 0.05 wage-share percentage points define distinct conditional breakdown questions; neither is a statistical significance threshold.

response lies in $(4, 32.0001]$ share points. The corresponding bracket for a response at most 0.05 wage-share points is $(1.75, 8.0001]$. These thresholds describe the precision of the conditional information experiment.

D.4 Signal resolution and revenue-response coefficients

The finer-bin feasible searches in Table D.11 cover the four years in which the primary search found negative responses. Each direction uses the same posterior starting allocation and a budget of 16 exact-gradient improvement iterations. The 20-bin controls match that budget. Search paths differ from the primary optimization, so their attained endpoints need not be the best primary endpoints. Negative attained values establish admissible reversals; positive attained minima leave other allocations unresolved.

Table D.12 records the annual response-coefficient distribution, location-floor mass, and concentration of squared response coefficients. Table D.13 separately changes common cost locations and

Table D.11: Attained finite responses with finer signal partitions

Year	Bins	Firms	Benchmark	Attained lower	Attained upper
2004	20	220,686	0.5280	-0.0304	1.0095
2004	40	220,686	0.5280	0.0142	0.9744
2004	80	220,686	0.5280	0.0658	0.9317
2005	20	218,358	0.4881	-0.0112	0.9330
2005	40	218,358	0.4881	0.0309	0.9012
2005	80	218,358	0.4881	0.0772	0.8631
2006	20	243,969	0.4422	-0.0334	0.8383
2006	40	243,969	0.4422	0.0068	0.8103
2006	80	243,969	0.4422	0.0572	0.7738
2007	20	274,443	0.3619	-0.0690	0.7319
2007	40	274,443	0.3619	-0.0386	0.7012
2007	80	274,443	0.3619	0.0007	0.6680

Note: This post-primary sensitivity exercise covers the four years with negative witnesses in the primary 20-bin search. Responses are percentage-point changes after the same 5% wedge relief. All retained records and response coefficients are unchanged. Only the signal partitions are refined; actual 20–40–80 nesting is verified with ties unsplit. No group expenditure, wage, exposure, or removal-benchmark moments are added. Each direction starts at the benchmark posterior and uses a fixed budget of 16 feasible-search iterations; no outer optimization is performed. Negative attained values establish feasible witnesses. A nonnegative attained lower value does not exclude a negative response. The extrema remain computationally unresolved.

Table D.12: Revenue-response coefficients and fitted-location boundaries

Year	Median λ	95th pct.	Max.	Floor mass (%)	$\mu \leq 1.02$ (%)	Top 1% share (%)
1998	3.39	8.37	18.70	0.0000	0.000	31.3
1999	3.57	7.40	26.37	0.0000	0.010	31.1
2000	3.35	8.80	15.03	0.0000	0.000	31.4
2001	3.55	9.23	21.84	0.0000	0.000	31.9
2002	3.17	9.28	19.39	0.0000	0.021	31.1
2003	2.91	9.51	29.10	0.0000	0.431	41.1
2004	3.09	10.95	42.60	0.0009	0.015	42.1
2005	3.14	10.46	34.02	0.0000	0.000	44.2
2006	2.94	10.00	42.67	0.0000	0.647	47.0
2007	2.52	9.14	22.07	0.0000	0.144	39.8

Note: Quantiles weight groups by posterior fractional value added. Floor mass uses $|\mu - 1.01| \leq 10^{-8}$. The final column is the fraction of $\sum_g \beta_g \lambda_g^2$ contributed by the largest 1% of groups ranked by that contribution, rounding the number of groups upward. Unavailable single-record α values are excluded from parameter distributions, with coverage recorded in the supplement; those records remain in economic aggregates and have uniform exposure, hence zero counterfactual response. Curvature depends jointly on these response coefficients and within-industry firm-level wage-share dispersion.

clips only near-boundary markups. The common shift indexed by f replaces μ_g by $\mu_g f/1.01$, preserving log differences and assignments. Clipping replaces μ_g by $\max\{\mu_g, 1.02\}$. Both are conditional parameter comparisons with fixed input parameters and records.

Table D.13: Markup Sensitivity with the Fitted Mixture Held Fixed

Year	Shift to 1.005	Original 1.01	Shift to 1.02	Clip at 1.02
1998	0.8631	0.8230	0.7552	0.8230
1999	0.8238	0.7844	0.7182	0.7842
2000	0.6872	0.6542	0.5985	0.6542
2001	0.6723	0.6369	0.5782	0.6369
2002	0.5689	0.5392	0.4900	0.5391
2003	0.5328	0.4999	0.4475	0.4979
2004	0.5684	0.5280	0.4660	0.5279
2005	0.5233	0.4881	0.4328	0.4881
2006	0.4834	0.4422	0.3877	0.4369
2007	0.3870	0.3619	0.3229	0.3619

Note: Benchmark wage-share effects of a 5% reduction in recipients' capital wedges, in percentage points. A shift indexed by f replaces every μ_g by $\mu_g f/1.01$, preserving log-location differences and posterior membership probabilities. The transformation is a common log-location shift indexed by a minimum markup, with the fitted mixture held fixed. Clipping replaces μ_g by $\max(\mu_g, 1.02)$ and changes only locations below that threshold. Clipping at 1.005 or 1.01 leaves all locations unchanged. α and data are held fixed.

Table D.14: Concentration and fractional assignments in finite counterfactuals, 2007

Assignment and state	Top 1% (%)	Avg. group max. (%)	Group HHI	Fractional firms	Fractional VA (%)
Benchmark baseline	39.34	11.48	0.0407	270,237	96.34
Benchmark counterfactual	39.71	11.68	0.0414	270,237	96.34
Direct finite lower witness	39.80	13.34	0.0527	13,313	12.48
Direct finite upper witness	39.47	13.15	0.0517	12,465	12.09
Fixed exposure share: lower	39.89	12.91	0.0486	12,661	18.09
Fixed exposure share: upper	39.64	12.99	0.0494	12,691	16.60

Note: The initial allocation and finite counterfactuals for 5% capital-wedge relief use all retained records. Firm-level revenue first sums that record across all fractional types; the top one percent is ranked across the sample. Group maxima and Herfindahl indices are weighted by initial group value added. A record is fractional when its largest assignment is below $1 - 10^{-8}$. The first two rows hold the assignment fixed. The remaining rows use assignments attaining lower or upper finite-objective values; comparisons with the baseline combine assignment differences and the intervention. The initial sample-wide top-one-percent value-added share is 39.34%.

Table D.15: Indivisible assignments under rounded nest-count constraints

Year	Firms	Lower attained	Upper attained	Max. quota change	Inactive groups
1998	112,707	0.2909	1.2163	0.652	0
1999	112,808	0.2630	1.1587	0.661	0
2000	115,724	0.1564	1.0399	0.658	0
2001	125,592	0.1125	1.0596	0.648	0
2002	134,447	0.1045	0.8724	0.714	0
2003	152,595	0.0523	0.8780	0.723	0
2004	220,686	-0.0231	0.9889	0.718	0
2005	218,358	-0.0020	0.9086	0.670	0
2006	243,969	-0.0270	0.8252	0.713	0
2007	274,443	-0.0659	0.7187	0.701	0

Note: Every retained firm has one group label. Within-bin nest-count constraints use fixed largest-remainder rounding and differ from the original posterior masses by less than one firm. Wage-share changes are percentage points; quota changes are firm-count units. These are attained outcomes of a discrete search with changed margins, under the stated integer margins; global discrete extrema remain unresolved.

D.5 Concentration, indivisible records, and shock size

For sample-wide concentration, all fractional types belonging to a record are aggregated before ranking. Group concentration is evaluated separately and weighted by initial group value added. The first two rows of Table D.14 compare the same benchmark assignment before and after the intervention. Its average group Herfindahl index rises from 0.0407 to 0.0414, and its average largest-firm share rises from 11.48 to 11.68 percent. The remaining rows describe counterfactual economies with different nest assignments as well as the wedge reduction; their concentration levels therefore combine the composition of those nests and the intervention. The sample-wide top-one-percent shares remain close across the displayed economies. These diagnostics describe the concentration associated with the maintained monopolistic-competition model.

The integer exercise in Table D.15 gives each retained record one label. Bin quotas are obtained by largest-remainder rounding of posterior masses, with ties broken by class index; every quota is its floor or ceiling and the bin total is exact. A maximum-affinity assignment and assignments based on saved local witnesses initialize the searches. Each direction uses at most eight exact-gradient transportation vertices, accepts only strict improvements, and retains the same stopping rule across signs. Every saved witness satisfies binary assignment and rounded quotas. The changed margins define a separate discrete problem. Negative feasible responses remain present in 2004–2007.

Table D.16: Alternative capital-wedge relief sizes in 2007

Relief	Benchmark	Attained range	Direct/Taylor outer	Lower gap	Upper gap
1%	0.071	[-0.016, 0.138]	[-0.074, 0.210]	0.058	0.072
5%	0.362	[-0.073, 0.733]	[-0.338, 1.117]	0.264	0.384
10%	0.733	[-0.140, 1.582]	[-0.612, 2.407]	0.471	0.825

Note: Identical retained firms, coefficients, fixed exposure and 20-bin capacities. Each shock is optimized separately under the common resource rule. All changes and gaps are percentage points.

Table D.16 retains the one-, five-, and ten-percent wedge-relief experiments on the same 2007 records. Each shock uses the common optimization resource rule. At the 2005 lower primary witness, the five-percent local approximation is -0.02446 points and the exact finite response is -0.01926 points, so that negative finite value also has a negative local response. Numerical validity, retained branch bounds, and resource limits are documented in Appendix B.

E Matched Reports and Recipient Composition

This appendix documents the matched sample, source-specific information sets, recipient accounting, and numerical verification for Section 6.

E.1 Matched Sample and Fixed Baseline Accounts

The starting pool consists of all contemporaneous dual-report matches in the eligible 2007 industrial manufacturing sample. Industrial cleaning applies the four log-ratio screens using the full annual industry sample. The additional source-specific sales-to-cost screens use the common matched pool. Their intersection retains identical records for the industrial and tax estimates. Values strictly outside the fixed first and ninety-ninth percentiles are deleted. The resulting 51,959 firms in 423 harmonized four-digit industries are the matched sample used for both recipient rules and reporting sources.

Table E.1 gives the matching and cleaning chain. The final matched sample retains 91.7965 percent of value added in the pre-tail matched pool. Relative to the complete primary retained industrial sample of 274,443 firms in 425 industries, the same final matched sample accounts for 18.9325 percent of firms, 40.5165 percent of value added, 40.6316 percent of wages, and 47.0257 percent of capital. These two coverage comparisons use different denominators: the matched pool describes cleaning losses within the source comparison, and the primary retained sample connects this application to the main annual sample. Matching uses contemporaneous reports.

Both reporting systems retain industrial value added V_i, wage payments W_i, capital K_i, employment N_i, industry, and record order. The shared industry labor output elasticities are estimated once on this common sample with the maintained two-piece-normal restriction and capital valuation. Each source separately applies the same four-digit-industry mixture procedure, component ceiling, and regularization to its own log sales-to-main-business-cost ratio. These estimates define the benchmark economies for the matched-sample counterfactuals. Collapsing equal component locations uses a 10^{-8} tolerance; posterior rows are renormalized only for floating-point roundoff, with a maximum change of 4.44×10^{-16}.

One industry contains a singleton with an unavailable finite input-location estimate. Its value added is 0.0001236 percent of the common total. Strict-above-median exposure is zero for this firm under both recipient rules, so its finite response is exactly zero for every $\alpha \in [0, 1]$. Its value added and wages remain in every denominator. Boundary-supremum input estimates are also retained: they cover 83 industries, 1,183 firms, and 4.6403 percent of common value added. Both reporting models and the shared labor output elasticities are mapped in all 423 industries.

E.2 Source-Specific Sets of Admissible Nest Assignments

Let $j \in \{I, T\}$ denote the industrial or tax reporting system, with benchmark posterior membership probabilities P^j_{ig} and response coefficient

$$\lambda^j_g = \frac{1 - \alpha_{s(g)}}{\mu^j_g - 1}.$$

The industrial and tax systems contain 751 and 719 benchmark nests, respectively. Each source defines up to twenty quantile bins of its own log sales-to-cost ratio within an industry; tied ratio values remain together. If B^j_b is one such bin, the benchmark group mass in firm-count units is $m^j_{bg} = \sum_{i \in B^j_b} P^j_{ig}$. Nests and bins are indexed within their industry throughout.

Table E.1: Matched-Sample Construction and Economic Coverage

Sample	Firms	Industries	VA retained (%)	Wages retained (%)
Same-year dual-report matches	56,506	423	100.00	100.00
Industrial four-ratio cleaning	52,668	423	92.91	94.12
Final common sample	51,959	423	91.80	92.98

Note: Coverage columns use the 56,506-firm same-year matched sample as the denominator. Matches originate in the 295,593-firm eligible industrial manufacturing sample. Industrial ratio screens use the full annual industry sample; source-specific sales/cost screens use the common pre-tail matched pool. The final matched sample is identical in both reporting systems. Relative to the complete primary retained industrial sample of 274,443 firms, final firm and value-added coverage are 18.9325 and 40.5165 percent.

For recipient rule $r \in \{KV, KN\}$, the exposure indicator e_i^r is computed once from the industrial accounts and reused for both sources. The ratio is strictly above its unweighted median within the common-sample four-digit industry, with ties unexposed. All 51,959 firms have positive reported employment, so the K/N rule uses the same support as K/V. The benchmark exposure share is

$$p_g^{jr} = \frac{\sum_i P_{ig}^j V_i e_i^r}{\sum_i P_{ig}^j V_i}.$$

The conditional set for source j and rule r contains fractional nest assignments satisfying

$$0 \le a_{ig}^j \le 1, \qquad \sum_g a_{ig}^j = 1,$$
$$\sum_{i \in B_b^j} a_{ig}^j = m_{bg}^j,$$
$$\sum_i a_{ig}^j V_i (e_i^r - p_g^{jr}) = 0.$$

The last equality fixes the exposed share of nest value added; total nest value added need not be supplied separately. All nest exposure equalities are retained, including the last nest; one redundant group-count equation per bin can be recovered from the row sums. The benchmark assignment is feasible by construction.

The four rows of Table 4 optimize the same finite wage-share formula with $\tau = -\log(0.95)$ over these four conditional sets. Industrial accounts, the recipient indicator within each rule, and the shared technology estimate are common. Source changes affect the cost-ratio bins, group masses within bins, exposure shares, and response coefficients. Each interval therefore conditions on its own reporting system. Comparing the industrial and tax results retains these distinct benchmark nesting structures and measures the sensitivity of the same intervention to the two benchmark economies.

E.3 A Conditional Comparison of Recipient Rules

For a fixed reporting source j, let $\mathcal{A}_j^{KV}$ and $\mathcal{A}_j^{KN}$ denote the two sets just defined. They have the same firms, bins, group masses within bins, response coefficients, and baseline payments. Their exposure restrictions are generated by different recipient indicators. Maintaining both restrictions defines

$$\mathcal{A}_j^{\cap} = \mathcal{A}_j^{KV} \cap \mathcal{A}_j^{KN}.$$

The same posterior P^j belongs to both sets because it generates both benchmark exposure-share vectors. This establishes nonemptiness. Each assignment in $\mathcal{A}_j^{\cap}$ represents a common baseline economy on which to evaluate the two interventions separately.

Write $\Delta_j^r(a)$ for the finite response under rule r and source j, and let $\underline{\Delta}_j^r$ and $\overline{\Delta}_j^r$ be its reported lower and upper outer endpoints. For every $a \in \mathcal{A}_j^{\cap}$,

$$\underline{\Delta}_j^{KV} - \overline{\Delta}_j^{KN} \;\le\; \Delta_j^{KV}(a) - \Delta_j^{KN}(a) \;\le\; \overline{\Delta}_j^{KV} - \underline{\Delta}_j^{KN}. \tag{E.1}$$

Both inequalities follow by applying the two recipient-specific bounds to the same admissible assignment. Using unrounded outer endpoints gives lower bounds of 0.2207747751 and 0.3051822089 percentage points under the industrial and tax reporting systems. The resulting outer intervals, rounded

Table E.2: Overlap of the Two Recipient Rules

K/V rule	K/N rule	Firms	VA share (%)	W/V (%)	K/N index
Unexposed	Unexposed	18,901	21.19	20.49	19.35
Unexposed	Exposed	7,185	30.70	10.72	94.66
Exposed	Unexposed	7,185	3.13	45.52	28.51
Exposed	Exposed	18,688	44.98	20.97	168.61

Note: Industrial reports on all 51,959 common records. The capital-per-worker index is the ratio of subgroup capital to subgroup employment divided by the same ratio for the entire common sample, times 100. Both capital and employment retain their original measurement definitions. No extra trimming or sample restriction is applied.

outward to four decimals, are $[0.2207, 0.8141]$ and $[0.3051, 1.0170]$. These bounds inherit the valid endpoint inequalities from two separate optimization problems. Optimizing the difference directly over the common set could tighten them.

The conditional ranking holds separately within each reporting system, at its fixed benchmark coefficients and under both of its exposure-share vectors. It compares wage-share responses to the same proportional reduction in the capital wedge under two recipient rules. The rules reach equal numbers of firms and different capital and value-added totals. An equal-budget comparison would require a specified fiscal cost and a corresponding normalization of the interventions.

E.4 Recipient Overlap and Within-Nest Accounting

Table E.2 reports the full cross-classification of recipient rules. The 18,688 firms exposed under both rules account for 44.98 percent of value added, while the 18,901 exposed under neither account for 21.19 percent. The two switching cells have equal counts but sharply different economic weights. Together they account for 33.8333 percent of value added. Table E.3 supplements the main composition panel with each subgroup's shares of total wages, employment, and capital, and its capital-per-worker index. Every ratio uses sums of the same industrial accounts; per-worker indices set the common-sample aggregate to 100.

For each source and recipient rule, define $M_g = \sum_i P^j_{ig} V_i$, $U_g = \sum_i P^j_{ig} V_i e^r_i$, $L_g = \sum_i P^j_{ig} W_i$, and $C_g = \sum_i P^j_{ig} W_i e^r_i$. The exact contribution in the main table is $k_g(C_g - p_g L_g)/T$, evaluated using the same benchmark assignment and response coefficients as the finite effect; k_g is the finite-change reweighting factor. The excess wage bill $C_g - p_g L_g$ compares recipients' wage payments with their share of nest value added multiplied by the nest wage bill. Main Table 5 sums positive and negative contributions separately; Appendix Table E.3 reports the corresponding group counts and the value-added shares associated with positive and negative excess wage bills.

Under K/V, positive excess wage bills occur in nests accounting for 95.35 percent of value added in the industrial benchmark and 96.05 percent of value added in the tax benchmark. Under K/N, negative excess wage bills occur in nests accounting for 72.91 and 73.86 percent, respectively. These weights describe the within-nest pattern underlying the signed sums, whereas the main composition panel describes the whole recipient sample. The accounting identities $K/V = (K/N)/(V/N)$ and $W/V = (W/N)/(V/N)$ hold record by record; their maximum numerical discrepancy is 7.11×10^{-15}. Cross-classification firm shares and value-added shares each sum to one, and the signed group sums reproduce all four benchmark responses within 10^{-12} in wage-share units.

Table E.3: Recipient Characteristics and Within-Nest Accounting

Panel A. Shares of Matched-Sample Totals and Capital per Worker

Rule	Status	Wages (%)	Employment (%)	Capital (%)	K/N index
K/V	Unexposed	41.27	40.82	17.59	43.08
K/V	Exposed	58.73	59.18	82.41	139.26
K/N	Unexposed	31.20	40.35	8.94	22.16
K/N	Exposed	68.80	59.65	91.06	152.66

Panel B. Within-Nest Contributions and Excess Wage Bills

		Nest contributions			VA shares by sign (%)	
Rule	Source	Positive	Negative	Near zero	Positive	Negative
K/V	Industrial	663	78	10	95.35	4.65
K/V	Tax	624	88	7	96.05	3.91
K/N	Industrial	192	549	10	27.09	72.91
K/N	Tax	178	533	8	26.10	73.86

Note: Panel A reports ratios of subgroup totals to common-sample totals; the capital-per-worker index divides subgroup K/N by full-sample K/N and multiplies by 100. Panel B counts positive and negative contributions using a 10^{-14} tolerance in wage-share units. The final two columns sum M_g/T over nests with $(C_g - p_g L_g)/T$ above or below the same tolerance. Remaining nests lie within the tolerance. Contribution signs and excess-wage-bill signs are calculated separately because the finite-change reweighting factor can be zero. The underlying industrial and tax systems contain 751 and 719 benchmark nests.

E.5 Numerical Verification of the Matched-Sample Counterfactuals

Each source-rule case covers all 423 industries, giving 1,692 industry cases. Fixed exposure shares make their objectives linear in nest assignments. The implementation solves the two endpoints, reconstructs the original exponential-tilt target at each attained assignment, and restores equality-dual bounds with bounded-variable reduced-cost corrections and an outward adjustment for floating-point error. Every case completes successfully. The displayed endpoints are the restored-dual outer endpoints; their largest separation from the attained feasible value is 1.64×10^{-11} in wage-share units, or 1.64×10^{-9} percentage points.

A separate reconstruction checks the saved inputs, 3,384 attained assignments, and 2,132 nontrivial dual certificates. Maximum row, bin-count, and variable-bound residuals are below 9.0×10^{-10}, and the maximum fixed-exposure-share residual is 3.44×10^{-11}. The linear and original finite objectives agree within 5.41×10^{-13}; reconstruction of the saved attained objective differs by at most 8.89×10^{-16}. Restored outer endpoints are independently recovered within 1.03×10^{-14}. These tolerances support the numerically sharp LP interpretation used in the main table. The bounds quantify uncertainty about nest assignments conditional on the benchmark coefficients and estimated group masses.

F Benchmark and Mapping Checks

This appendix explains the removal benchmark used in the information comparison and the economic mappings underlying the benchmark response coefficients.

F.1 Information in the Wedge-Removal Benchmark

After removing both revenue and capital wedges, the wage share within nest g is α_s/μ_g. For an admissible assignment with nest value added M_g, the aggregate endpoint is

$$LS^{\text{rem}}(a) = \frac{1}{T}\sum_g M_g(a)\frac{\alpha_{s(g)}}{\mu_g}. \tag{F.1}$$

At fixed parameters, preserving each industry's value-added-weighted inverse markup preserves its removal benchmark. The 2007 comparison in Table D.4 imposes this linear restriction at the benchmark value. Its negative and positive feasible finite responses show that this benchmark leaves relevant associations between recipient status and firm-level wage shares unrestricted. Fixing nest value added is a stronger restriction on that same benchmark. Fixed exposure-share information addresses the finite target through a different statistic.

F.2 Economic Mapping and Revenue-Response Coefficient Ranges

Equation (C.7) allows deviations in the cost-ratio component means $|\delta_g| \le d_C$ and a deviation in the input-ratio normalization $|\nu_s| \le d_I$. At $r = r_0$, the primitive ranges are

$$\mu_g^- = \max\{1.01, \hat{\mu}_g e^{-d_C}\}, \qquad \mu_g^+ = \hat{\mu}_g e^{d_C}, \qquad \alpha_s^- = \text{logit}^{-1}(c_s - d_I), \qquad \alpha_s^+ = \text{logit}^{-1}(c_s + d_I).$$

Since $\lambda_g = (1-\alpha_s)/(\mu_g - 1)$, these ranges imply

$$\lambda_g^- = \frac{1-\alpha_s^+}{\mu_g^+ - 1}, \qquad \lambda_g^+ = \frac{1-\alpha_s^-}{\mu_g^- - 1}. \tag{F.2}$$

The smaller radii are $(d_C, d_I) = (0.01, 0.05)$ and the larger radii are $(0.025, 0.10)$. The nonlinear denominator means that these log-location radii can produce substantially different relative changes in revenue-response coefficients across nests. The Cartesian product of these coefficient intervals contains every admissible parameter configuration with a common labor output elasticity within each industry. The rectangle also permits vectors that combine different technology endpoints across nests. Feasible outcomes under common industry technology use one common α_s in each industry, admissible nest markups, and assignments satisfying the stated moment restrictions.

The input-location estimator is translation equivariant. Replacing wages by aW and the rental normalization by br shifts its center by $\log a - \log b$, giving

$$\hat{\alpha}_s(a,b) = \text{logit}^{-1}(\hat{c}_s + \log a - \log b).$$

Capital valuation and the total input discrepancy enter the same location index. Table F.1 reports annual estimates under alternative wage-and-benefit measures and rental normalizations, together with the frequency of boundary fits. The wages-plus-benefits specification is reestimated on common reporting support because the additional benefits vary across firms.

F.3 Uniform Tail Treatment and the Wedge-Removal Benchmark

Table F.2 compares the same grouped-minus-pooled removal statistic under untrimmed, trimmed, and winsorized inputs. Each treatment reestimates all mixture candidates and input parameters on its complete annual sample. The winsorized reconstruction uses the same ratio cutoffs as trimming

Table F.1: Labor Output Elasticities under Alternative Payment and Capital Normalizations

Year	$W, r = .2$	$W + B, r = .2$	$W, r = .1$	$W, r = .3$	Boundary industries	Boundary VA (%)
1998	0.332	0.357	0.487	0.254	47	0.82
1999	0.344	0.367	0.498	0.265	45	0.93
2000	0.369	0.394	0.527	0.286	34	0.35
2001	0.389	0.412	0.546	0.305	37	0.74
2002	0.411	0.436	0.569	0.325	35	0.47
2003	0.436	0.462	0.594	0.348	30	0.35
2004	0.473	0.498	0.629	0.383	24	0.35
2005	0.484	0.508	0.639	0.392	24	0.29
2006	0.511	0.535	0.665	0.418	20	0.26
2007	0.537	0.561	0.689	0.444	14	0.12

Note: Labor output elasticities are weighted by value added. The $W + B$ specification adds accrued employee welfare benefits B and uses the firms reporting nonnegative benefits. Rental coefficients refer to the supplied capital units. Boundary estimates are half-normal suprema. Constant samples receive ranges (smaller than displayed precision); numerical status and all contribution bounds are in the replication files.

and retains all eligible firms. Thus the comparison records how the benchmark economic mapping changes with the treatment of unusual reported ratios.

Table F.2: The grouping increment under uniform tail treatments

Year	Modal untrimmed	Modal trimmed	Modal winsorized	Posterior untrimmed	Posterior trimmed	Posterior winsorized
1998	0.375	0.260	0.265	0.035	-0.051	-0.044
1999	0.456	0.243	0.309	0.087	-0.071	-0.009
2000	0.379	0.184	0.217	-0.014	-0.138	-0.096
2001	0.367	0.182	0.214	-0.039	-0.141	-0.111
2002	0.373	0.173	0.239	-0.062	-0.158	-0.114
2003	0.342	0.096	0.052	-0.163	-0.289	-0.308
2004	0.284	-0.096	-0.037	-0.311	-0.517	-0.453
2005	0.584	0.225	0.296	-0.036	-0.171	-0.143
2006	0.595	0.280	0.356	-0.011	-0.162	-0.101
2007	0.500	0.082	0.140	-0.195	-0.408	-0.354

Note: Each entry is a grouped-minus-pooled wage-share difference in percentage points. Each treatment reestimates the labor output elasticity and mixture parameters on its annual sample. The untrimmed specification uses the eligible sample before tail treatment. Winsorization retains those firms and reconstructs input and cost levels from the clipped ratios.

Posterior and modal assignments weight the same benchmark nest parameters differently. The posterior increment is negative in every trimmed annual sample and in every winsorized annual sample, while the modal comparison also retains its negative 2004 values. These are conditional benchmark comparisons for the stated treatment. The finite-response results use the primary trimmed specification throughout.

F.4 Alternative Capital Measures on Identical Firms

The supplied accumulation series and average annual net fixed-assets balance give two observed capital measures. Their comparison uses all positive-book-capital firms, retaining 99.163–99.977 percent

Table F.3: Capital Measurement and the Removal Wage-Share Benchmark

Year	VA retained (%)	Scale c_t	Benchmark using original α	Refit using supplied K	Refit using book K	Refit using scaled book K
1998	99.728	1.857	27.199	27.181	36.524	25.431
1999	99.748	1.757	28.275	28.286	36.429	26.331
2000	99.163	1.659	30.288	30.270	37.454	28.223
2001	99.534	1.585	32.137	32.135	38.346	29.851
2002	99.750	1.511	34.008	33.993	39.154	31.517
2003	99.646	1.432	36.150	36.122	40.201	33.450
2004	99.972	1.332	39.555	39.560	43.194	37.774
2005	99.977	1.269	40.755	40.755	42.950	38.361
2006	99.977	1.186	42.956	42.958	44.533	41.226
2007	99.901	1.129	44.738	44.745	45.580	43.251

Note: The last four columns are posterior-weighted wage shares after removing revenue and capital wedges, in percent, on the identical positive-book-capital subset. Fitted component means and posterior membership probabilities remain fixed. Original α carries the full-sample estimate onto that subset; the next two columns reestimate it using supplied capital and average annual net fixed-assets book balances, respectively. The final column multiplies book capital by $c_t = \sum_i K_i / \sum_i K_i^{book}$ before estimating α; the translation property supplies the identical result without another fit. The rental coefficient 0.2 is the common normalization in all log input ratios. Equal aggregate capital defines the scale comparison; an economic user-cost comparison additionally requires compatible valuation and price information. Degenerate-input contributions use their lower endpoint; their maximum interval width is 0.000418 wage-share percentage points. Numerical diagnostics preserve all boundary fits and the two book-capital fits slightly above the numerical gap tolerance.

of annual primary value added. Fitted cost-ratio component means, posterior membership probabilities, wages, and value added stay fixed. Input parameters are reestimated from $\log[W_i/(0.2K_i)]$ and $\log[W_i/(0.2K_i^{\text{book}})]$. Carrying the full-sample input parameters onto the subset separately measures the composition change.

We also scale every book-capital value within year by $c_t = \sum_i K_i / \sum_i K_i^{\text{book}}$. This preserves relative capital stocks across firms within each year and matches aggregate book capital to the supplied series. Translation equivariance shifts the book-capital center by $-\log c_t$, supplying the scaled estimate without further optimization. The scaling equalizes aggregate measured capital; the rental coefficient remains a normalization per capital unit.

The removal benchmark serves here as a common scalar diagnostic of the input mapping. On identical 1998 firms it is 27.181 percent under reestimated supplied capital, 36.524 under book capital, and 25.431 under aggregate-scaled book capital. The corresponding 2007 values are 44.745, 45.580, and 43.251 percent. These differences show the sensitivity of mapped input parameters to both an annual scale and the relative capital distribution. The comparison assesses sensitivity of the input mapping to capital measurement.

All numerical statuses remain in the comparison. Constant-input industries in 2000–2001 have no finite likelihood estimate; their $\alpha \in [0,1]$ contribution changes posterior endpoints by at most 0.000418 points. Half-normal boundary fits are retained. One book-capital fit in each of 2006 and 2007 has a per-observation gap slightly above 10^{-9}, approximately 1.04×10^{-9} and 1.17×10^{-9}. These cases remain in Table F.3 and its recorded diagnostics.